USE OF CAPACITORS IN BRAKING OF LARGE INDUCTION MOTORS

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ABSTRACT

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The use of capacitors in braking of large induction motors results in considerable braking torque produced by the self excitation action. The objective of this paper is to explain the theory, different design methods, sizing of the capacitors, speed torque relations, motor losses and industrial applications of capacitor braking method.

Furthermore the braking effect can be achieved by capacitors alone or in combination with series or parallel resistors.

Braking Torque can be also controlled by a rectifier inverter combination which acts as a variable secondary resistance. Also, braking effect is produced by connecting equal or unequal capacitors with series or parallel resistors in the secondary side of the motor. When the load inertia exceeds six times that of the motor, capacitor and magnetic braking with controlling resistors is used.

In cases when load inertia is higher, capacitor braking followed by the simultaneous use of magnetic and d.c. injection braking is used.

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LIST OF SYMBOLS

V_c	Capacitor voltage, volts.
f	Frequency of excitation/frequency of supply.
I_1	Primary current, amps.
I_2	Secondary current, amps
I_m	Magnetising current, amps.
N_o	Synchronous speed, radians/second
r_1	Per-phase resistance of primary winding, ohms.
r_2	Per-phase resistance of secondary winding, ohms
s	Slip
N	Speed, radians/second
T	Torque, Newton-metre
V	Line voltage, volts
x_1	Per-phase primary winding reactance, ohms
x_2	Per-phase secondary winding reactance, ohms.
x_c	Capacitive reactance, ohms.
θ	Invertor firing angle, degrees
E	Airgap induced voltage, volts
C	Equivilant capacitance/phase, Farad
ϕ	Flux
P	Number of poles
n	Speed/Synchronous speed.

V_1, $V_1 \underline{/240^o}$, $V_1 \underline{/120^o}$ = Line voltages

I_1, I_2, I_3 = Line currents.

I_a, I_b, I_c = Phase currents

V_p, V_n = Positive and negative sequence components respectively of the voltage applied to the windings.

I_p, I_n = Positive and negative-sequence components respectively of the stator phase currents

I_{2p}, I_{2n} = Positive and negative-sequence components respectively of the rotor phase currents

Z_p Z_n = Impedance of the motor per phase, to positive- and negative sequence currents respectively

Z_c = Capacitive reactance introduced in one line.

Z_r = Inductive reactance introduced in another line

The subscripts d and q designated d- and q-axis variables referred to the stationary reference frame. The prime (') designates the value of motor rotor quantities referred to the stator. The subscript a designates phase a instantaneous voltage or current variables.

v_{as}, v_{qs}, v_{ds} = stator and capacitor terminal voltages

v_{aL}, v_{qL}, v_{dL} = voltages of infinite bus

$v'_{aL}, v'_{bL}, v'_{cL}$ = voltages of motor bus

i_{as}, i_{ds}, i_{qs} = stator currents

$i'_{ar}, i'_{qr}, i'_{dr}$ = rotor currents

i_{ac}, i_{qc}, i_{dc}	=	capacitor currents
i_{aL}, i_{qL}, i_{dL}	=	line currents supplied by distribution system
w_e	=	radian frequency of distribution system
θ_r	=	angle between phase a axes of rotor and stator
w_r	=	$p\theta_1$ = angular velocity of rotor
p	=	time derivative operator, d/dt
Xc	=	$1/\hat{w}_e C$ = reactance per phase of shunt capacitor bank at distribution system frequency
r_L, L_L	=	resistance and inductance per phase of distribution system
N_s, N_c	=	series and common winding turns per phase of ideal autotransformer
a	=	$(N_a + N_c)/N_c$ ratio of transformation
r_s, s'_r	=	motor resistances per phase
L_{ss}, L'_{rr}	=	unsaturated values of apparent self-inductance per phase of stator and rotor when carrying balanced three-phase currents
X_{ss}, X'_{rr}	=	reactances of the foregoing inductances evaluated at frequency w_e
X_{1s}, X'_{1r}	=	leakage reactances of stator and rotor at frequency w_e
$Xm = X_{ss} - X_{1s} = X'_{rr} - X'_{1s}$	=	unsaturated value of magnetizing inductance per phase

T_L	=	mechanical load torque
T_d	=	motor developed torque
H	=	combined inertia constant of motor and load
δ	=	instantaneous phase difference of voltages to be synchronized
K_v	=	voltage ratio tolerance for synchronizing

1.0 INTRODUCTION

For modern high speed machinery, one of the major problems facing the control engineer in designing the electric drives is that of stopping the machine in the shortest possible time in order to comply with certain prescribed limits of position for a process or product being made. Braking of the induction motor is a control function which dissipates the kinetic energy of the motor and any connected load during the period of (deceleration) of the motor speed from a high value to either a lower value or zero.

The dissipation of this kinetic energy may be either external to the motor shaft or internal in the motor windings. Comparison and discussion of different methods of the induction motor braking will be explained in detail in section 1.2

Static capacitors are used for the braking of three phase Induction motor. Braking torque is obtained by generator action when the motor is disconnected from the line and excitation current is supplied by capacitors connected across the line.

By introducing capacitors to the primary side of the motor, Self excitation takes place through the capacitors which produce discharge excitation current.

This current produces its own magnetic field that oppose the one already in the machine at the moment of disconnection from the supply.

This magnetic field built by the self excitation through the capacitors can be used to obtain high braking torques. These torques can be increased by dissipating part of the motor kinetic energy in the externally connected resistors.

The rate of decrease of capacitor voltage, excitation frequency and motor speed is directly proportional to the rate of decrease in kinetic energy.

TABLE I

1.1 INDUCTION MOTOR BRAKING METHODS (1)

Method of braking	I.M.	
	Single phase	Poly phase
Plugging, Counter torque	Repulsion Universal	Yes
Regenerative	Yes	Yes
Dynamic, AC excitation	No	Yes
" DC excitation	Most types	Yes
" DC with resistors	Universal only	Unicersal only
Capacitor excitation	No	Yes
Capacitor, resistor and Rectifier	Most types	Yes

1.2 Comparison of Braking Methods:- (1)

In this section comparison between the main two methods of braking the induction motor is taken place as follows:

1.2.1 External Braking (1)

Is provided by some form of a mechanical brake which is coupled to the motor shaft, the brake might be actuated mechanically, electrically, hydraulically or pneumatically with proper control elements.

Advantages of external braking: (1)

(1) Proper holding action

(2) Heating is not produced in the motor.

Disadvantages of external Braking: (1)

(1) Counter torque can not be provided.

(2) Space is required in addition to that required for the motor.

(3) Special shaft extension is required.

(4) Inertia is added to the motor rotating system by the brake rotating element.

(5) Kinetic energy is not recovered.

1.2.2 Internal Braking. (5)

Internal braking torque for deceleration is produced electrically by the currents flowing in the windings. The methods for producing internal electrical braking torque can be classified into two groups: counter torque and generating action and it is listed in the table provided in section 2.1.

Advantages of internal braking(5)

(1) Braking torque is developed internally in the machine.

(2) Inertia is not added to the rotating system

Disadvantages of internal braking(5)

(1) Braking torque disappears at zero or low speed.

(2) Heating is produced in the motor windings.

(3) Power is required for braking, except for capacitor braking.

(4) Currents magnitudes are in the same order as acceleration currents.

(5) Dissipation of heat decreases as speed decreases.

Further to the above comparison of three types of internal braking is discussed below. The braking types are:

(a) Plugging (5)

Plugging of a 3-phase squirrel-cage motor is accomplished by simply inter-changing any two of the supply lines. The surge of current in stator and rotor is somewhat more severe than at starting unless a reduced voltage is provided. For loads having only a small amount of friction torque, the energy which is dissipated as heat in the rotor in braking the load to standstill is nearly three times the kinetic energy of the motor and load. This extreme heating of the rotor would seem to limit the application of plugging of squirrel-cage motors to cases where either low inertia or infrequent stops would make it practical. An additional complication is the necessity of an automatic switch of some kind to remove power at the instant the motor stops, otherwise the motor and load will be brought quickly to full speed in the opposite direction. The magnitude of the braking torque can be controlled only by the application of an adjustable voltage during the braking period.

(b) Application of direct current to the motor: (5)

Application of direct current to one phase of the stator of a squirrel-cage motor will provide a braking torque which can be adjusted quite easily. Heating energy equal to the kinetic energy of the motor and load will be produced in the rotor for applications having a small friction torque. The braking torque will vary with the speed of the motor and in general will follow a characteristic similar to that shown in the diagram of Figure I.

The difficulty of dissipating any great amount of energy either in a mechanical brake or in the rotor of the squirrel-cage motor has led several engineers to investigate the possibility of operating a motor as an induction generator during the braking procedure and thereby make possible the transfer of power from the machine to an external resistor. Considerable study has been made of the operation of an induction machine with self-excitation supplied by static capacitors. Most of the published information, however, deals with operation at a constant frequency and little has been done on the varying frequency problem of dynamic braking.

(c) Induction generator action by means of capacitor excitation: (5)

The self-excitation is known to decrease rapidly with decrease in speed of the motor. However, if braking can be accomplished even to one-half speed, three-fourths of the kinetic energy of the rotating parts will have been recovered The application of the method of braking has been handicapped by the fact that no very simple or reliable method has been available for predetermining the speed-torque characteristic which will result from a given motor, capacitor, and resistor combination. The purpose of this report is to describe a method by which this speed-torque characteristic can be calculated with fair accuracy and with a minimum of test data concerning the motor. Typical speed-torque characteristics for the four methods of braking are shown in Figure 1.

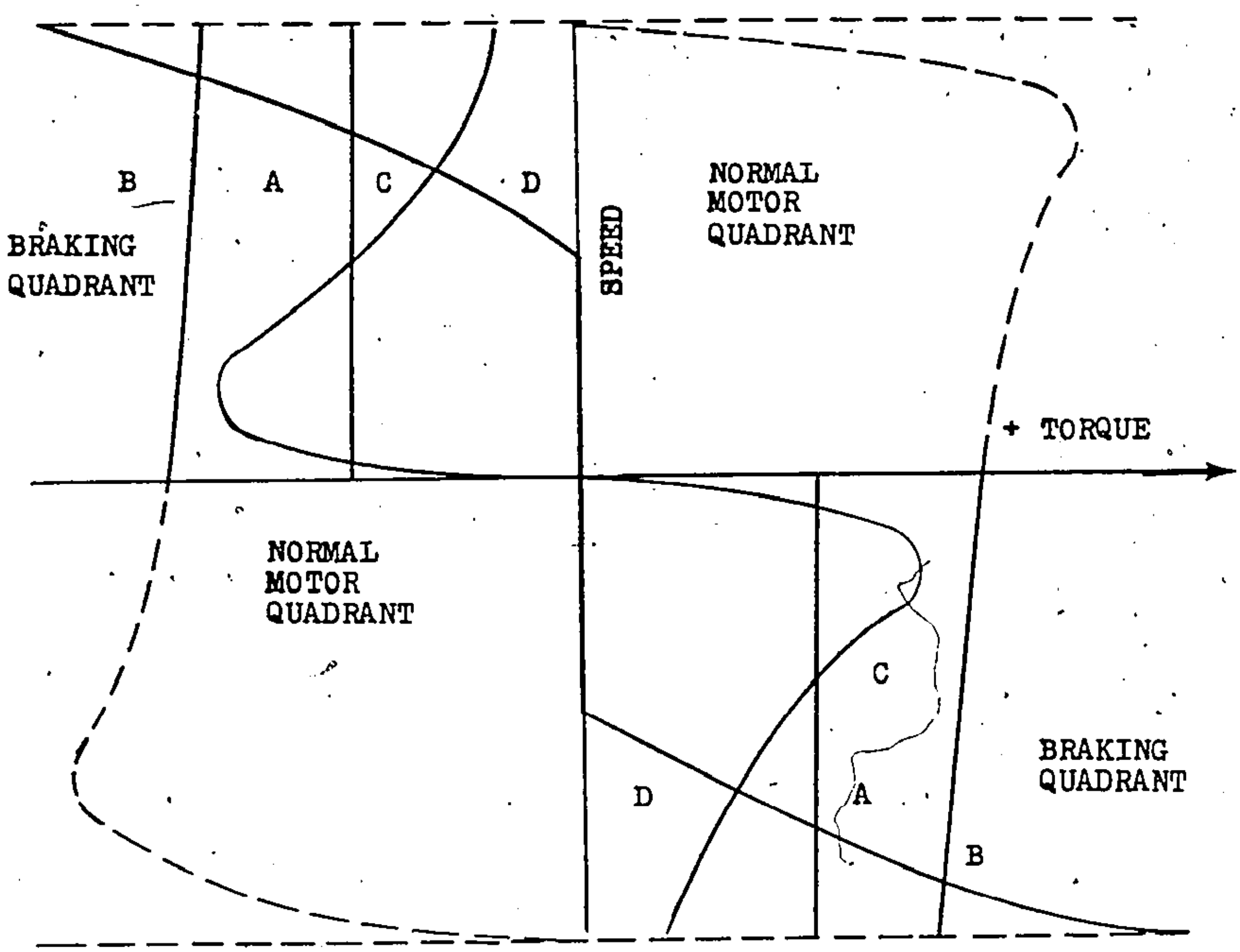


Figure 1. Typical speed-torque characteristics for several types of braking control

A - Mechanical brake

B - Plugging

C - Direct current applied to stator

D - Capacitive braking

2.3 Advantages of Capacitor braking

(1) Increased braking torque is developed.

(2) The system can be applied to existing machines

(3) The reduction in losses enables high inertia loads to be dealt with and the frequency of stopping to be increased compared with plugging (3).

(4) Consistently high accuracy of positioning during stopping can be obtained whenever capacitor braking with magnetic dynamic braking is used.

(5) The system has lower power cansumption than other methods.

(6) It is silent in operation.

(7) It gives a power factor correction to the motor when in normal motoring use.

(8) It is independent of supply voltage failure.

Disadvantages:

(1) Capacitor excitation falls rapidly with decrease of speed (3/4 of the drive kinetic energy has been dissipated by the time ½ speed have been achieved.)

1.4 Restrictions on Capacitor braking of Induction Motor

(1) Disconnection of induction motors from the supply can give rise to high transient torque on reswitching, if the disconnection period is relatively short. If capacitor-excitation occurred during the period of disconnection it has been found that the transient torque on reswitching can be relatively much larger compared to the case where no excitation took place. (2)

(2) The value of capacitance used must not exceed that needed to correct the motor no-load power factor to unity because damage to the winding insulation from high transient voltage would occur. (1)

2.0 RLC EQUIVALENT CIRCUIT

The equivalent circuit of an induction motor during capacitor braking is presented in section 3. (Fig.17)

The resonance effect and the response of the RLC series network will be related to the roots of the characteristic equation in the complex s plane. Applying Kirchhoff voltage law gives

$$L\frac{di}{dt} + Ri + \int\frac{1}{C}\ idt = 0 \qquad (i.1)$$

The corresponding homogeneous equation is of second order and is

$$\frac{d^2i}{dt^2} + \frac{R}{L}\ \frac{di}{dt}\ + \frac{1}{LC}\ \ i = 0 \qquad (i.2)$$

The two roots of the corresponding characteristic equation may be found by the quadratic formula to be

$$S_1,\ S_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \qquad (i.3)$$

To convert eq. (i.2) to a standard form, we define the value of resistance that causes the radical term in eq. (i.3) to vanish as the critical resistance, Rcr. This value is found by solving the equation $\left(\frac{Rcr}{2L}\right)^2 = \frac{1}{LC}$ (i.4) or $Rcr = 2\sqrt{\frac{L}{C}}$ (i.5)

Defining: Damping ratio $\zeta = \frac{R}{Rcr} = \frac{R}{2}\ \sqrt{\frac{C}{L}}$ (i.6)

is the ratio of the actual resistance to the critical value of resistance.

Defining: Undamped natural frequency $W_n = \frac{1}{\sqrt{LC}}$ (i.7)

The product $2\zeta W_n = R/L$ (i.8)

Substituting in eq. (i.2) gives $\frac{di^2}{dt^2} + 2\zeta Wn\,\frac{di}{dt} + W_n{}^2 i = 0$ (i.9)

or $S^2 + 2\zeta W_n S + Wn^2 = 0$ (i.10)

The roots of the characteristics equation are:

$$S1,\ S2 = -\zeta W_n \pm W_n \sqrt{\zeta^2 - 1} \qquad (i.11)$$

The damping ratio ζ varies from zero (corresponding to $R = 0$) to infinity (corresponding to $R = \infty$)

Defining Self excitation frequency Fex = $\frac{wex}{2}$ is the driver frequency of the system.

Roots on the imaginary axis correspond to oscillatory response (zero damping). As the value of the damping coefficient is a function of the resistance, which in terms is variable depending on the slip $\left(\frac{R_2}{S}\right)$, so zero damping condition is not to be considered in this RLC circuit as well as the resonance effect.

3.0 BRAKING ANALYSIS

Different design methods describing the use of capacitors in braking of the induction motor are explained in this section.

Also, the theory for capacitor excitation, speed-torque characteristics, losses during braking, the effect of motor magnetic saturation, braking time and temperature rise and speed time relations are described.

3.1 Dynamic Braking by Capacitors in the supply line to the Induction Motor (1)

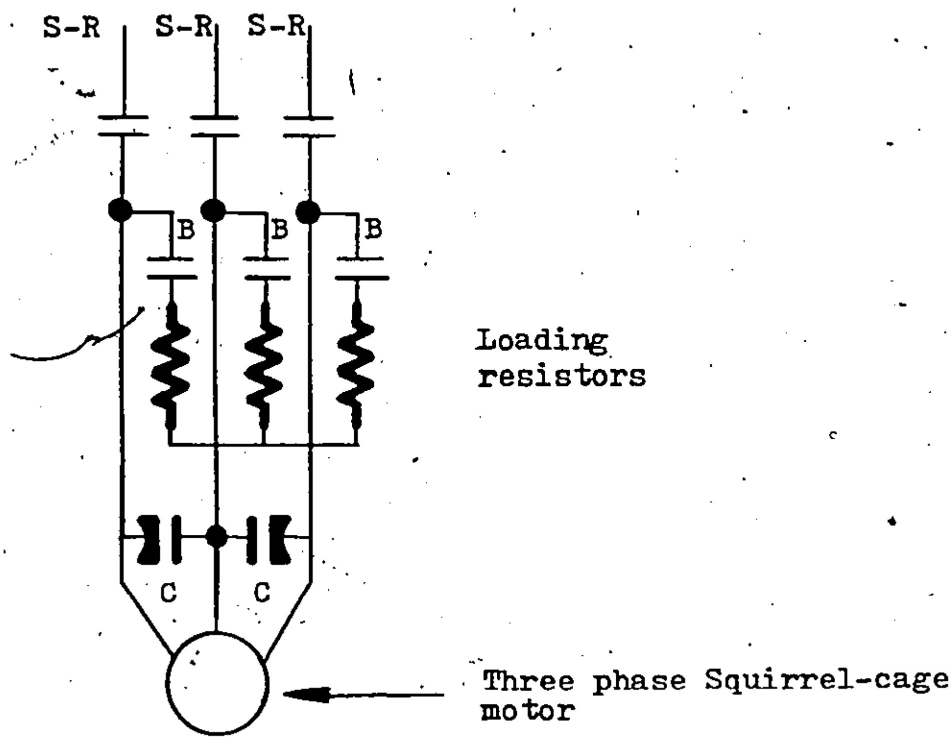


Fig.2 Dynamic braking by capacitors

S:	Start	B:	brake
R:	Run	C:	Capacitor

As stated earlier, dynamic braking torque is obtained by generation action when the motor is disconnected from the supply and loading excitation current is supplied by capacitors connected across the line .

The braking torque can be increased by using external resistors as a load, to dissipate part of the motor kinetic energy.

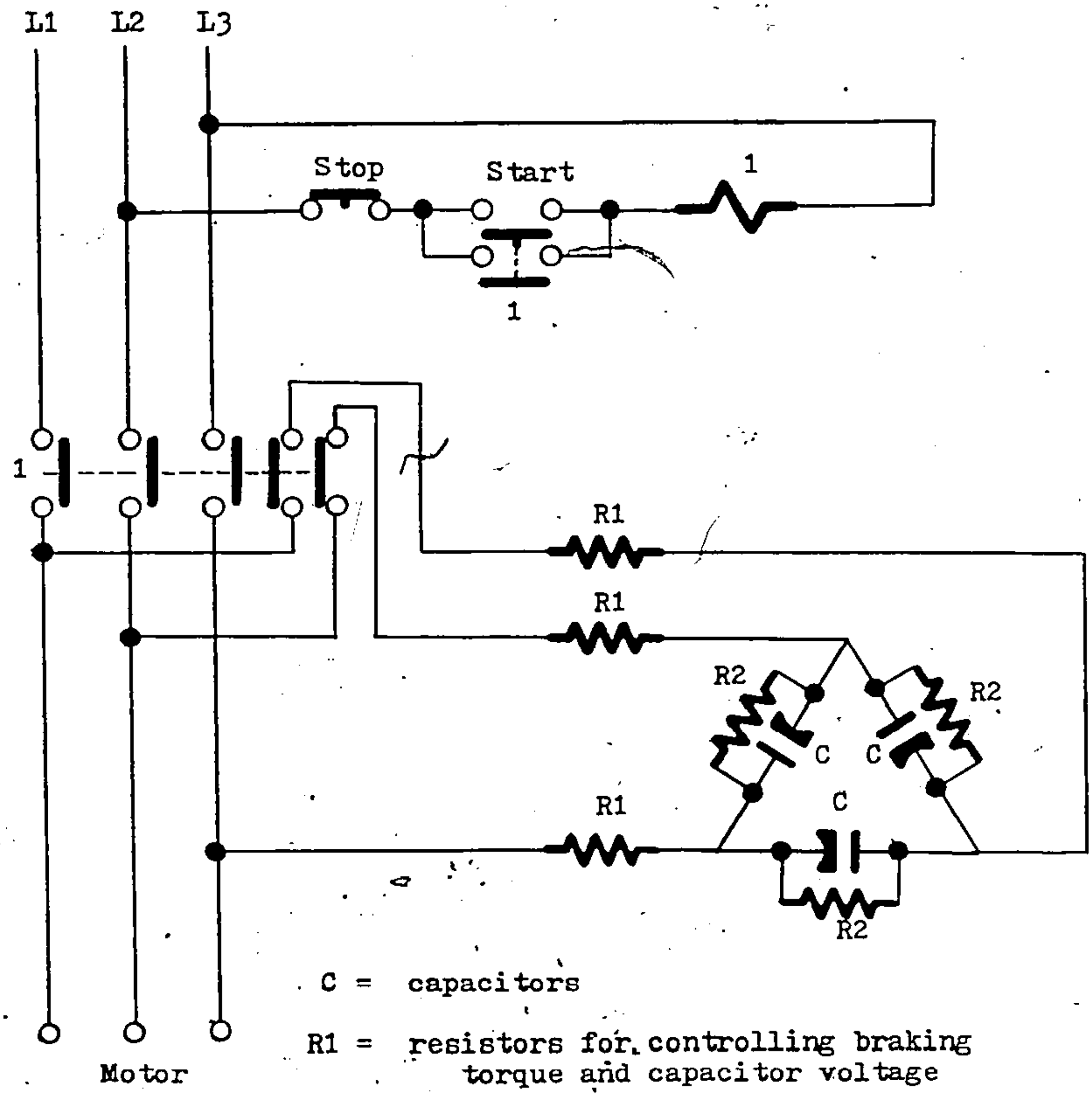
When the capacitors are connected across the line, the braking torque produced is not dependent upon the power system voltage. It decreases following the curve D fig.1, dropping to zero at approximately 1/3 of motor initial speed.

Fig.2 shows the connection diagram for dynamic braking by capacitors connected across two phases. The value of capacitance used must not exceed that needed to correct the motor no load power factor to unity because damage to the winding insulation from high transient voltage could occure.

3.2 Use of a Capacitor only, with or without Controlling Resistors (3)

This method is used mainly where complete stoppage is not required and where load friction is enough to stop the motor in the required time after capacitor self-excitation has ceased.

Resistors in circuit with the motor windings may be used to control the torque and capacitor voltage, the connections being as shown in fig.3.



C = capacitors

R1 = resistors for controlling braking torque and capacitor voltage

R2 = discharge resistors

Fig.3 The basic connection diagram for capacitor dynamic braking.

Examples of applications for which this arrangement is suitable are: (a) the reduction of impact shock of a valve gate on the seating at the end of its travel, and (b) the braking of a moving part to about 10% of its original speed so that the energy which must be absorbed in a mechanical stop at a predetermined point can be kept low. An instance of this was the control of a trolley weighing 450lb in a tube making factory. Buffers to stop the trolley could withstand only a limited amount of shock, so dynamic braking was made to reduce the travelling speed of 740ft/min by 90% in 0.45 second. (3)

3.3 <u>Dynamic Braking by Capacitor-Rectifier-Resistor Combination</u> (1) Dynamic braking by capacitor-rectifier-resistor combination indicates that dynamic braking is obtained by generator action with a form of d.c excitation.

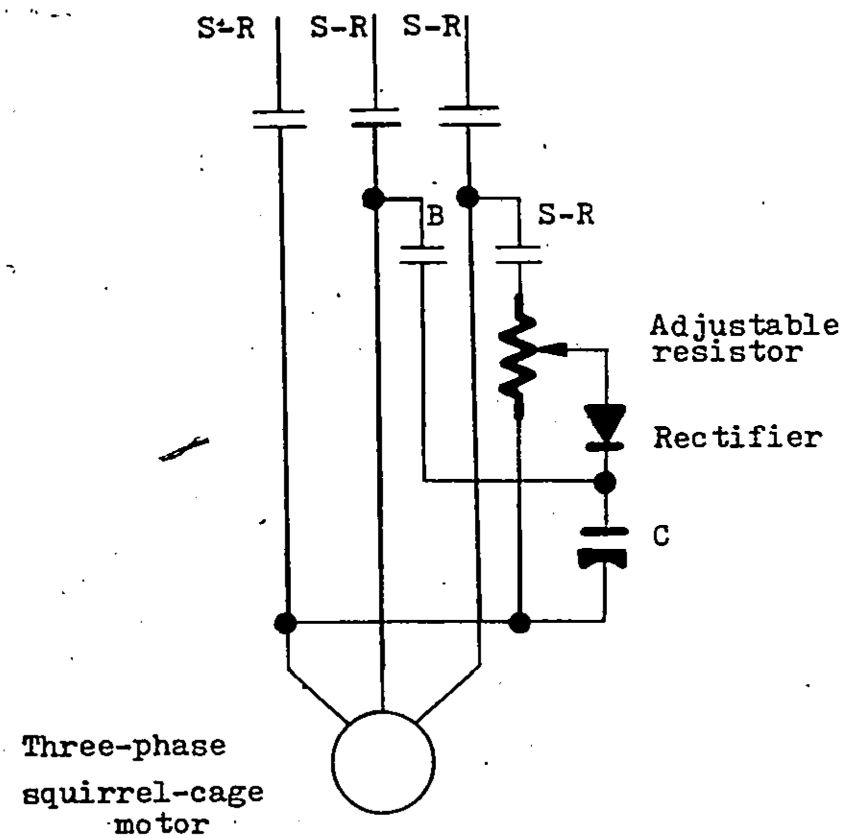
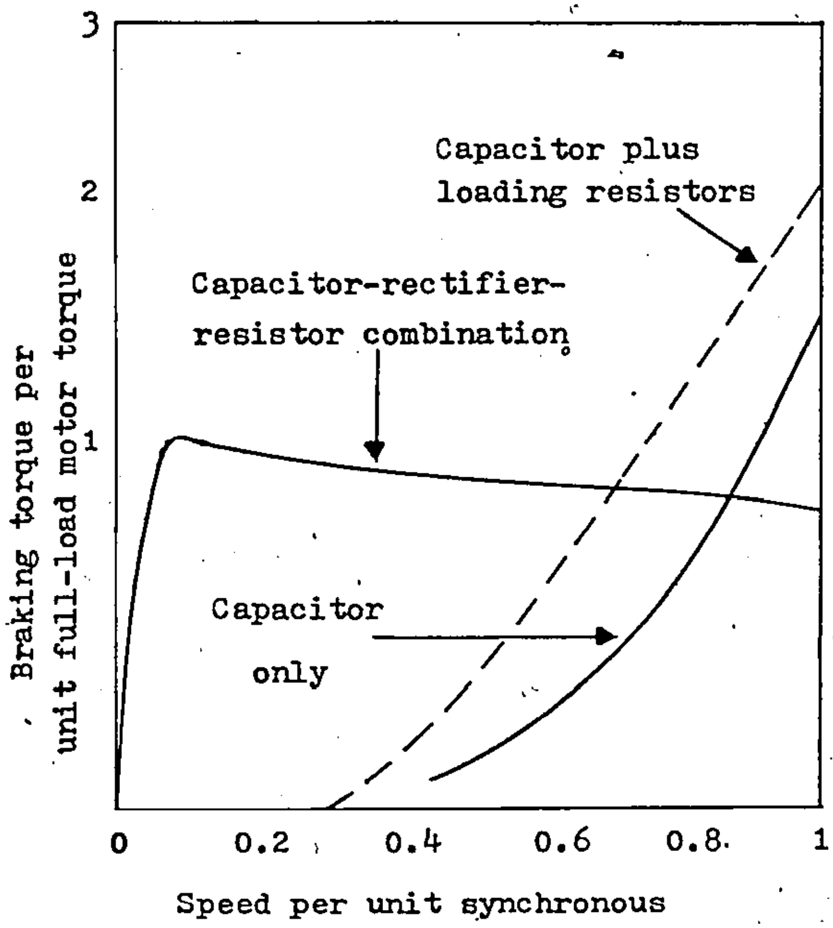


Fig. 4. (a)

S - Start   B - Brake

R - Run   C - Capacitor

Dynamic Braking by Capacitor- Rectifier Resistor Combination



(b)

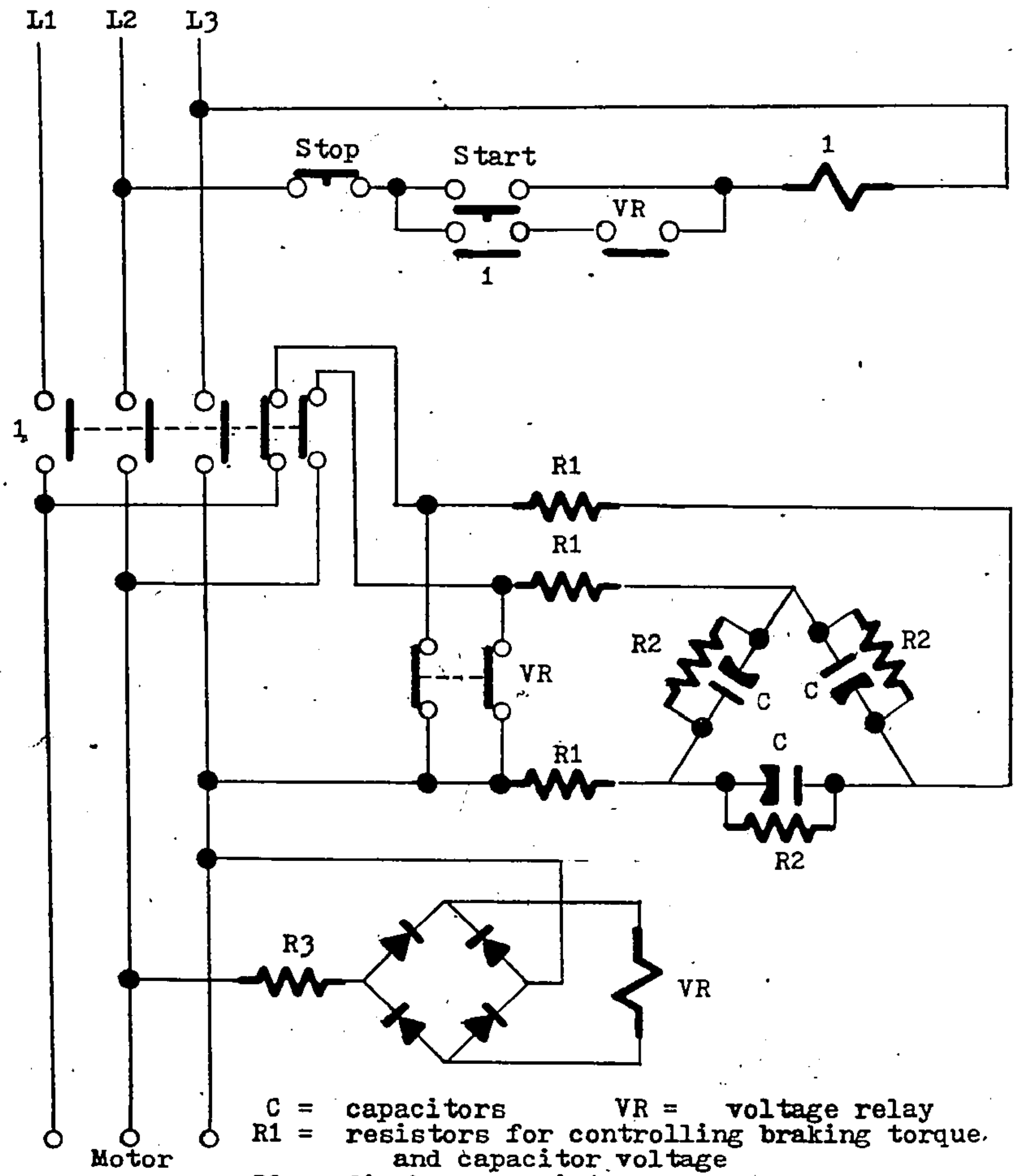
Fig.4 Dynamic braking by capacitors. Typical braking-torque curves are shown for dynamic braking by capacitors, for capacitors plus loading resistors, and for capacitor-rectifier-resistor combination.

A capacitor which is charged through a rectifier circuit when the motor is running discharges direct current through the motor windings to produce braking torque when the motor is disconnected from the line (Fig.4a). This method is applicable to three-phase induction motors. It differs from dynamic braking by capacitor in that dc power is discharged into the motor windings.

The capacitor discharge provides maximum voltage at maximum motor speed. The dc voltage and motor speed decrease together to give more nearly constant braking torque. The dc excitation is available until the capacitor has discharged. Capacitor values chosen to match the motor and load characteristics can result in discharge time equal to deceleration time. The energy stored in the capacitor is all that is required for the braking power supply. Large capacitor value are required. Figure 4b shows the braking torque typical of dynamic braking by the capacitor-rectifier-resistor method.

3.4 Use of Capacitor and Magnetic Braking with Controlling Resistors (3)

This is the most useful arrangement and is suitable for all cases except those where the load inertia exceeds about six times that of the motor. The circuit used, shown in fig.5 includes a voltage relay to operate at the moment



C = capacitors VR = voltage relay
R1 = resistors for controlling braking torque and capacitor voltage
R2 = discharge resistors
R3 = control resistor for relay drop out voltage

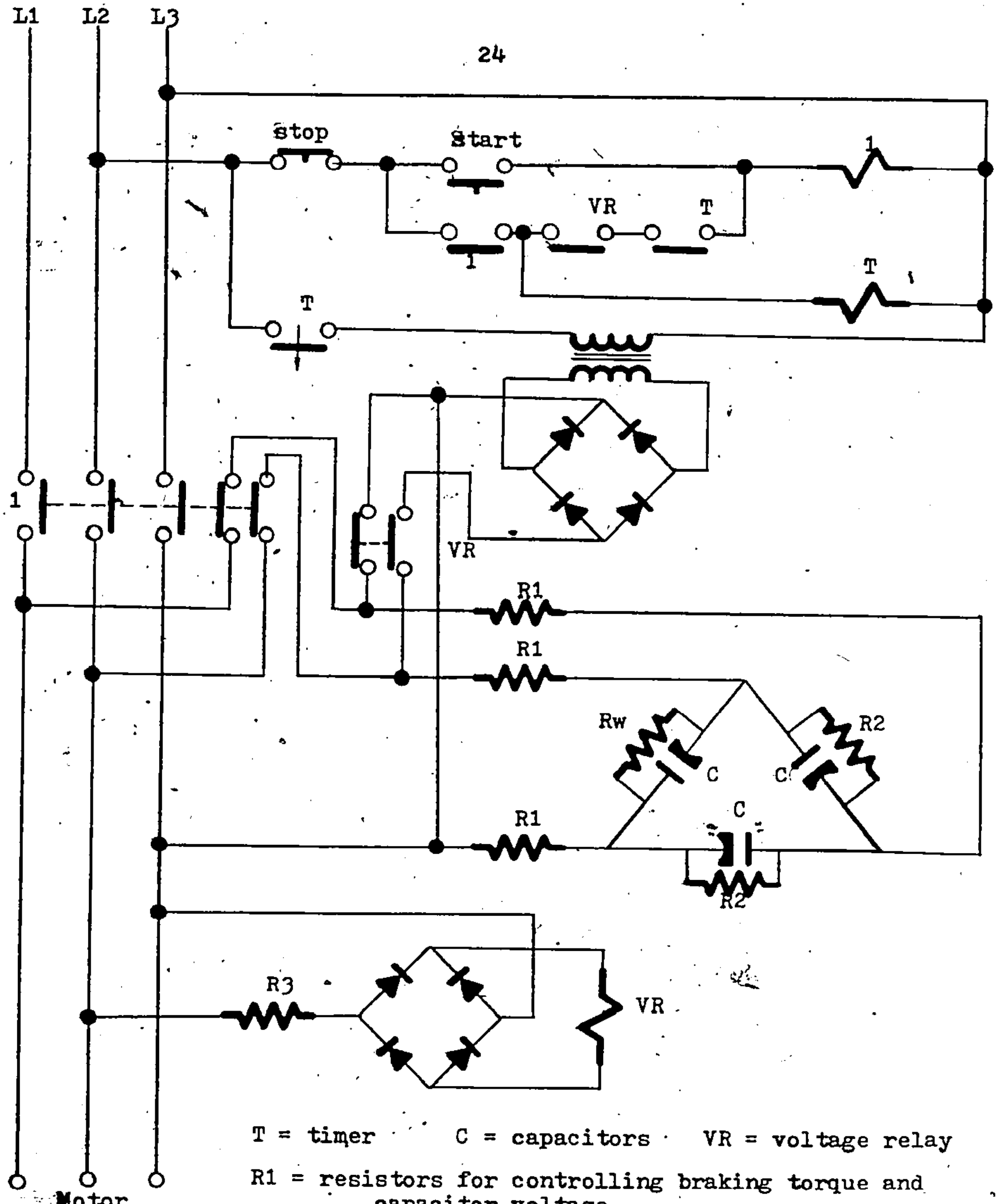
Fig.5 The basic connection diagram for capacitor and magnetic dynamic braking.

when the motor terminals must be short-circuited. In place of this relay a timer or limit switches may be used.

3.5 Use of Capacitor Braking Followed by the Simultaneous use of Magnetic and D.C. Injection Braking (3)

This method is used when the load inertia is higher than the previous method can handle. The direct current, which has only a small part to play in the scheme, is provided by the use of rectifiers supplied from the same source as the motor. The circuit is shown in fig. 6.

A typical application of this method was to a 25hp, 2 pole motor driving a large band saw, the motor inertia being 1.5lb ft^2 and the load inertia sixteen times that of the motor.



T = timer   C = capacitors   VR = voltage relay

R1 = resistors for controlling braking torque and capacitor voltage
R2 = discharge resistors
R3 = resistor for control line relay drop-out voltage

Fig.6 The basic connection diagram for a system giving capacitor braking followed by simultaneous magnetic and d.c. injection braking.

3.6 Braking of a static slip-power recovery drive. (2), (10)

It has been shown that the braking torque produced by an induction motor with balanced primary capacitor excitation can be controlled by the addition of external secondary resistance. Consider a conventional slip-ring induction motor, disconnected from the electrical supply, with equal primary side capacitance per phase, operating at some forward speed.

With zero additional secondary resistance there exists the familiar circuit for capacitor excitation of the induction motor and, with sufficient capacitance, a braking torque is produced. For the condition of infinite additional secondary resistance the secondary circuit is open-circuited, the machine fails to excite and no electrical braking torque is then produced. Between these two extremes of additional secondary resistance, control of the braking torque produced by an induction motor with balanced primary capacitor-excitation, at a given speed, is possible.

Computed results showed that an induction motor running at constant speed, with balanced primary capacitors providing excitation, would give reduced braking torque as the external secondary resistance is increased. The energy dissipated within the whole circuit is mainly in the primary windings for low values of additional secondary resistance but, for larger values of secondary resistance, the values of dissipated energy in the primary and secondary circuits become comparable.

An adjustable secondary "resistance" can be inserted into an induction motor secondary winding by use of the rectifier-invertor combination. This combination is used in the well-established static, slip-energy recovery system for induction motor speed control shown in Fig. 7.

With a diode bridge rectifier at the motor secondary terminals, feeding a naturally commutated thyristor invertor, Fig. 7, energy is extracted from the secondary windings and returned to the supply. Control is thereby achieved for increase of speed over the subsynchronous motoring region from zero to rated motor speed. The effect of the rectifier-invertor combination is equivalent to a form of slip-frequency secondary voltage injection, or an adjustable secondary resistance. The equivalent inserted "resistance" is a function of speed, secondary current and invertor firing angle.

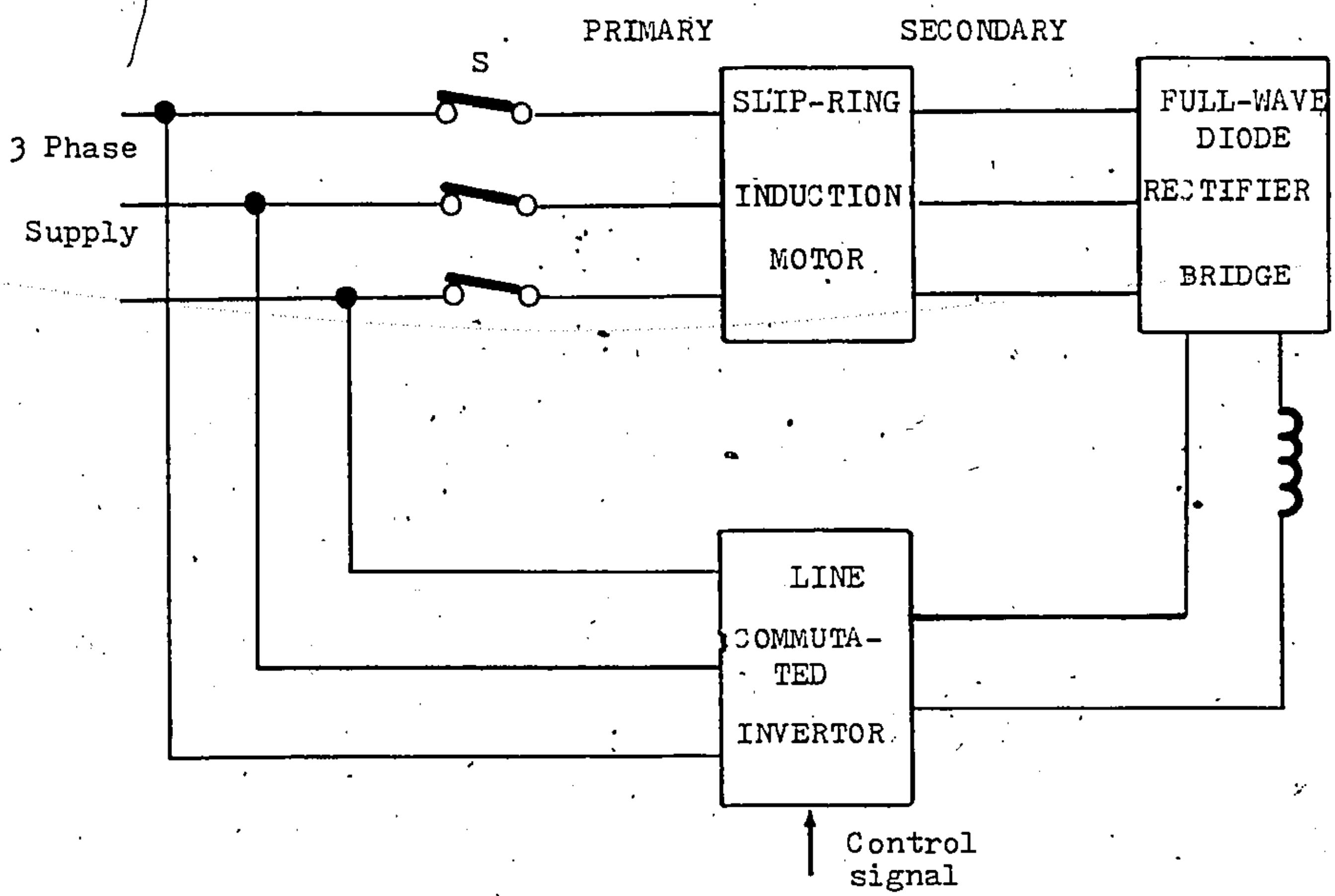


Fig. 7. Arrangement of slip-energy-recovery drive. (SER)

3.7 SYMMETRICAL COMPONENTS CONSIDERATIONS TO CAPACITOR BRAKING (6)

3.71. INTRODUCTION:

If the voltages applied to the terminals of a poly-phase motor are unbalanced, the unbalanced currents drawn by the motor can be resolved into a set of balanced positive-sequence components and another set of balanced negative-sequence components. The former exerts a torque in the positive direction (direction of rotation) and the latter, in the negative direction. A single-phase reactor is introduced in one of the 3-phase supply lines, thereby causing unbalanced voltages to be impressend on the motor terminals. Thus, if there is no asymmetry in the circuit, the current drawn is completely of positive sequence; if asymmetry is caused by the introduction of a single-phase impedance, the current consists of both positive-sequence and negative-sequence components, and if there is no asymmetry but two leads are interchanged (plugging), the current is completely of negative sequence.

Considering the case when asymmetry is caused by the introduction of a single-phase impedance, the magnitudes of the positive and negative-sequence components, the torque exerted by them, and the net torque developed by the motor depend upon the value of the impedance and the speed of the motor. If a reactor is used, the negative-sequence component will always be less than the positive-sequence component and the motor will always develop driving torque. If an infinite reactor is used (one line open) the negative and positive-sequence components of current are equal, but the motor still develops driving torque. If a capacitor is used, the negative-sequence component is greater than the positive-sequence component; but the net torque developed by the machine may be either driving torque or braking torque, depending upon the speed of the machine.

3.7.2 CURRENT AND TORQUE EQUATIONS (6)

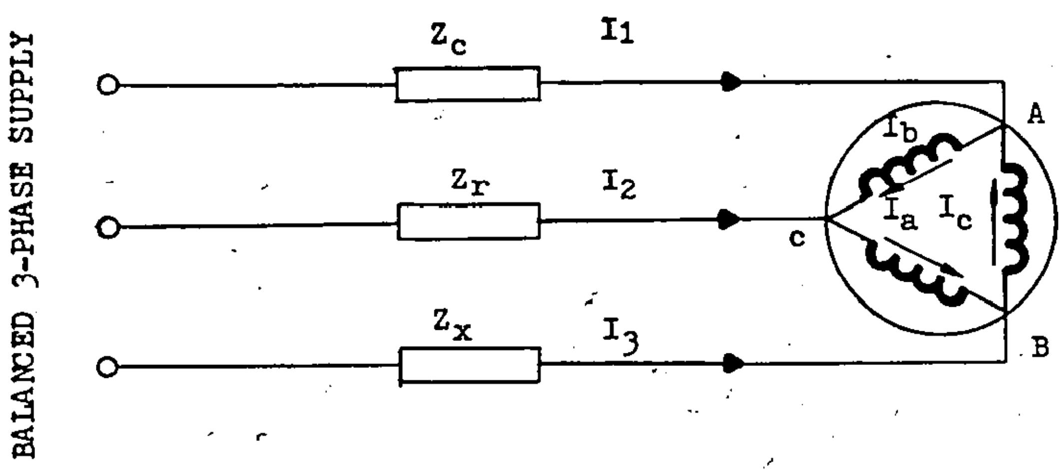


Fig.8. Motor connection pertaining to the general current and torque equations.

For a delta-connected motor (Fig.8), considering the general case of three series impedances Zc, Zr and Zx in the supply lines, the following equations are derived:

$$V_a = V_1 - I_2 Z_r + I_3 Z_x$$

$$V_b = V_1 \angle 240 - I_1 Z_c + I_2 Z_r$$

$$V_c = V_1 \angle 120 + I_1 Z_c - I_3 Z_x$$

$$V_p = 1/3\ (V_a + V_b \angle 120 + V_c \angle 240)$$

$$V_p = V_1 + \frac{I_1 Z_c \angle 90 - I_2 Z_r \angle 30 + I_3 Z_x \angle 30}{\sqrt{3}}$$

$$V_n = 1/3\ (V_a + V_b \angle 240 + V_c \angle 120)$$

$$V_n = \frac{I_1 Z_c \angle 90 - I_2 Z_r \angle 30 + I_3 Z_x \angle 30}{\sqrt{3}}$$

$$I_p = \frac{V_p}{Z_p} = \frac{V_1}{Z_p} + \frac{I_1 Z_c \angle 90 - I_2 Z_r \angle 30 + I_3 Z_x \angle 30}{\sqrt{3}\, Z_p} \qquad (1)$$

$$I_n = \frac{V_n}{Z_n} = \frac{I1\ Z_c \angle 90 - I_2 Z_r \angle 30 + 2_3 Z_x \angle 30}{\sqrt{3}\ Z_n} \qquad (2)$$

$$I_a = I_p + I_n$$

$$I_b = I_p \angle 240 + I_n \angle 120$$

$$I_c = I_p \angle 120 + I_n \angle 240$$

$$I_1 = I_b - I_c = \sqrt{3}\ \ (I_p \angle 90 + I_n \angle 90 \qquad (3)$$

$$I_2 = I_a - I_b = \sqrt{3}\ \ (I_p \angle 30 + I_n \angle 30) \qquad (4)$$

$$I_3 = I_c - I_a = \sqrt{3}\ \ (I_p \angle 150 - I_n \angle 30) \qquad (5)$$

Substituting these expression for $I_1 - I_3$ in the expression for I_p and I_n:

$$I_p = \frac{V_1 (Z_n + Z_c + Z_r + Z_x)}{Z_p Z_n + (Z_p + Z_n)(Z_c + Z_r + Z_x) + 3(Z_c Z_r + Z_r Z_x + Z_x Z_c)} \qquad (6)$$

$$I_n = \frac{V_1 (Z_c + Z_r \angle 240^\circ + Z_x \angle 120^\circ)}{Z_p\ Z_n + (Z_p + Z_n)\ (Z_c + Z_x) + 3(Z_c Z_r + Z_r Z_x + Z_x Z_c)} \qquad (7)$$

$$I_{2p} = I_p\ \frac{Z_m}{Z_m + \frac{R_2}{S} + jx_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (8)$$

$$I_{2n} = I_n \frac{Z_m}{Z_m + \frac{R_2'}{2 - S} + jx_2} \quad \ldots\ldots\ldots\ldots\ldots \quad (9)$$

R_2' is the rotor resistance to the negative-sequence component of rotor currents. In the numerical calculations it is assumed to be equal to R_2.

Hence, torque in at any slip S is given by

$$3I_{2p}^2 \frac{R_2}{S} - 3I_{2n}^2 \frac{R'_2}{2 - S} \quad \ldots\ldots\ldots\ldots \quad (10)$$

3.7.3 Capacitor Braking Utilizing a capacitor in one phase only (6)

If a capacitor Zc only is used in one of the lines putting Zr = 0 and Zx = 0 in eqns. (6) and (7) gives

$$I_p = \frac{V_1(Z_n + Z_c)}{Z_pZ_n + (Z_p + Z_n)Z_c}$$

$$I_n = \frac{V_1 Z_c}{Z_pZ_n + (Z_p + Z_n)Z_c}$$

$$I_a = \frac{V_1(Z_n + 2Z_c)}{Z_pZ_n + (Z_p + Z_n)Z_c}$$

$$I_b = \frac{V_1(Z_n + Z_c\angle 60^\circ)\ \angle 240^\circ}{Z_pZ_n + (Z_p + Z_n)\ Z_c}$$

$$I_c = \frac{V_1(Z_n + Z_c\angle 60^\circ)\ \angle 120^\circ}{Z_pZ_n + (Z_p + Z_n)\ Z_c}$$

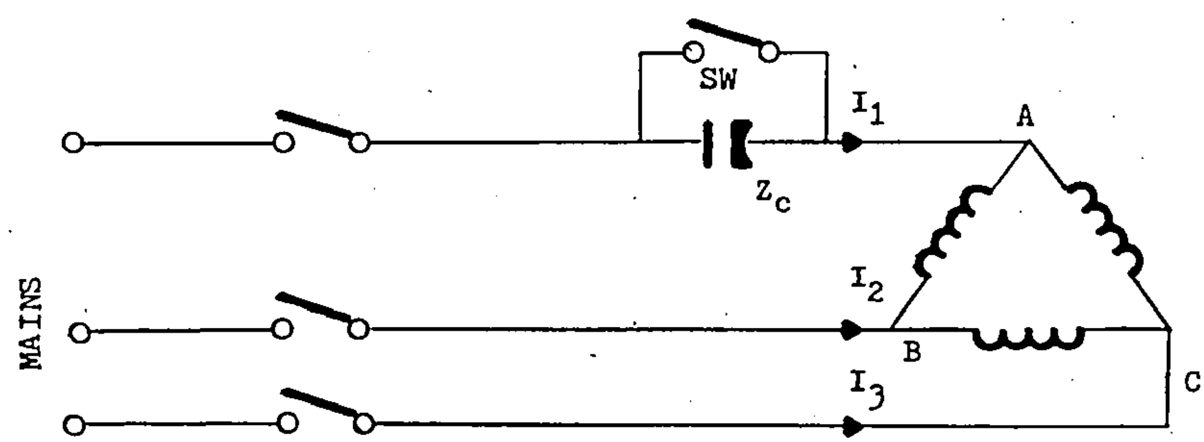


Fig.9 Capacitor braking utilizing capacitor in one phase SW open during braking period.

3.7.4 Capacitor-Reactor Braking (6)

If a capacitor, Z_c, is introduced in one line (Fig.10) and a reactor, Z_r, in another line, by making appropriate substitutions in eqns. (6) and (7) and by putting $Z_x = 0$, I_p, I_n and the speed/torque characteristic can be caulculated as in the previous case.

A graphical construction for I_p, I_n and torque in terms of the corresponding quantities under balanced conditions may also be adopted for capacitor braking. When the graphical method is extended to cover capacitor-reactor braking, it becomes too involved to be of practical use.

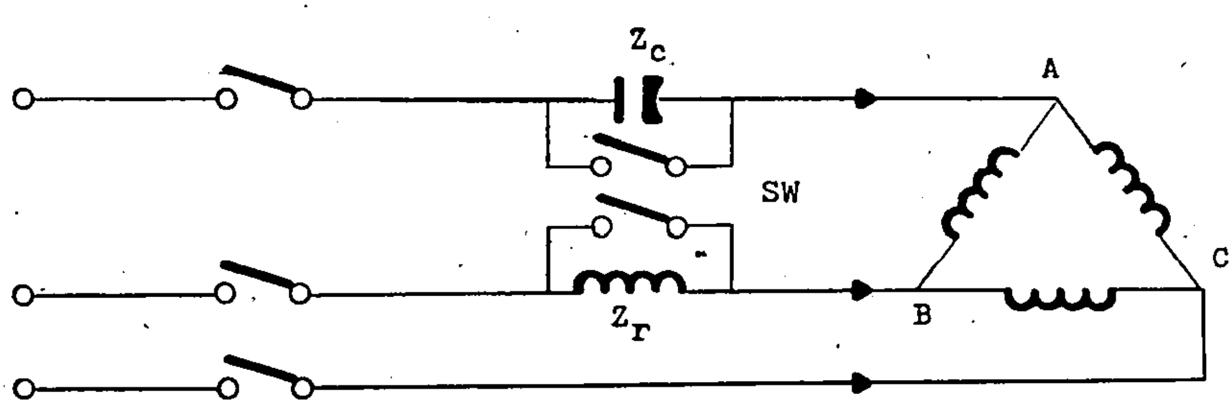


Fig.10. - Capacitor-reactor braking.
SW open during braking period.

3.7.5 Braking by Generator Action (6)

If the main switch is opened, especially during the initial stages when the speed is high, and connections are made as shown in Fig.11, the machine may work as a capacitor-excited Induction generator, wasting the kinetic energy of the revolving masses as heat in the stator and rotor windings. The generator action can only continue if the speed is above a certain value which depends upon the value of the capacitor used.

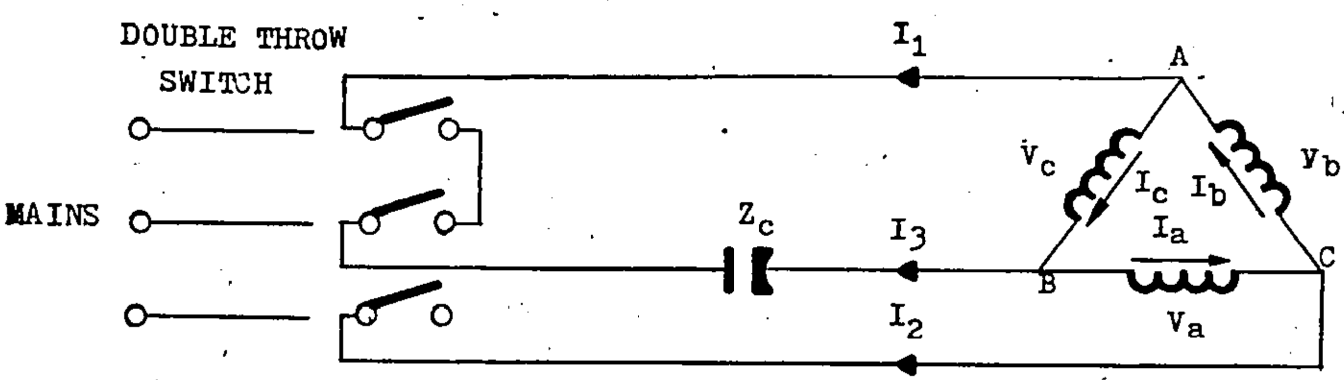


Fig. 11 - Braking by generator action

$$I_a = I_p + I_n = . \; I_b = I_p \angle 240^\circ + I_n \angle 120^\circ$$

Hence, $I_p = I_n \angle 120^\circ$

$$I_c = I_p \angle 120^\circ + I_n \angle 240^\circ$$

$$I_1 = I_c - I_b = I_p \angle 120^\circ + I_n \angle 240^\circ - I_p \angle 240^\circ$$
$$- I_n \angle 120^\circ = \sqrt{3} \angle 90^\circ (I_p - I_n) = 3I_p \angle 120^\circ$$

Hence, $I_p = \frac{I_1}{3} \angle 120^\circ$, $I_n = \frac{I_1}{3} \angle 240^\circ$

$$V_p = Z_p \, I_p = Z_p \frac{I_1}{3} \angle 120^\circ$$
$$V_n = Z_n \, I_n = Z_n \frac{I_1}{3} \angle 240^\circ$$

$$V_c = V_p \angle 120^\circ + V_n \angle 240^\circ = \frac{I_1}{3} (Z_p + Z_n)$$

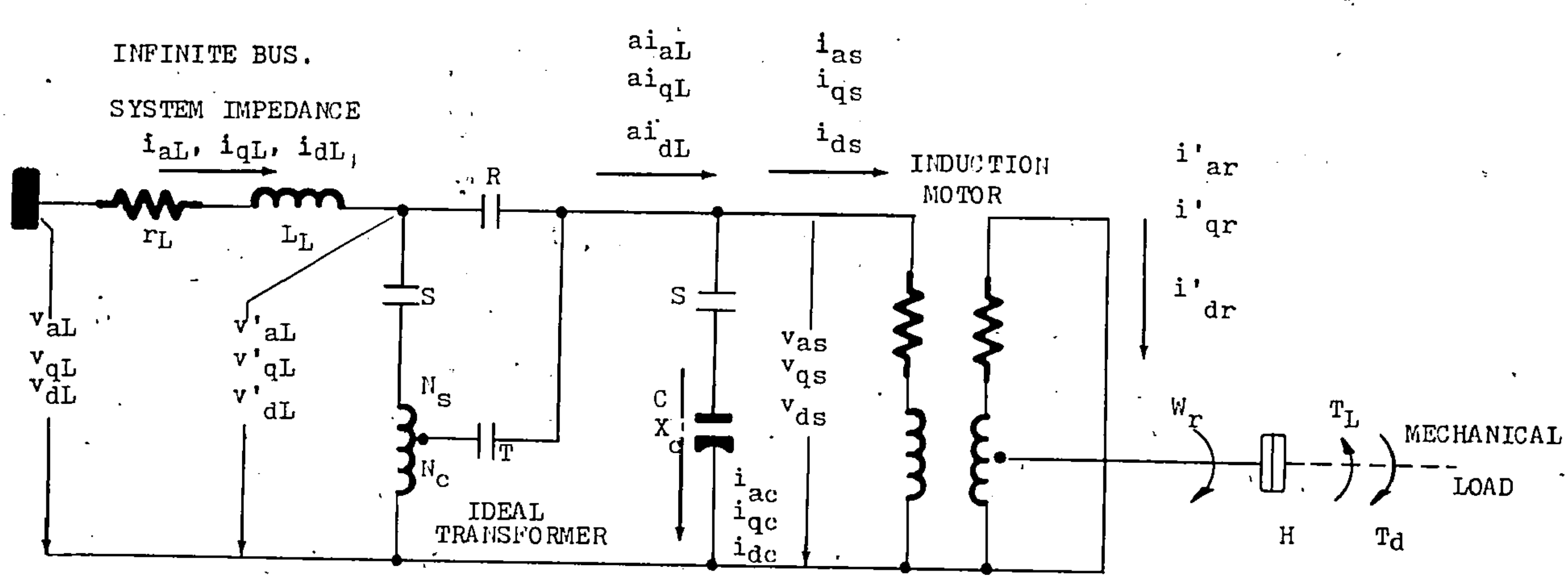
3.8 DYNAMIC ANALYSIS OF CAPACITOR BRAKING (9)

(a) System Studied

The system studied is that of Fig. 12 when contactors R and T are open and contactor S closed. The motor and its connected mechanical load are assumed to have been accelerated previously at reduced voltage, but are not coasting under the influence of the combined inertia constant of motor and load. Stator and capacitor currents, when they exist, are identical in magnitude.

The assumptions and idealizations made about the squirrel cage motor are:

1. three-wire, three=phase stator connections
2. zero core loss
3. uniform air gap
4. sinusoidal space distribution of rotor and stator MMF
5. sinusoidal variations of mutual inductances with rotor position
6. the magnetization curve of the machine displays saturation.



SUBSCRIPT a DESIGNATES PHASE a OF THE a-b-c VARIABLES. SUBSCRIPTS d AND q DESIGNATE d-q VARIABLES IN THE STATIONARY REFERENCE FRAME. THE PRIME (') DESIGNATES ROTOR QUANTITIES REFERRED TO THE STATOR.

CONTACTOR OPERATING SEQUENCE

STEP	CONTACTOR R	CONTACTOR S	CONTACTOR T
1	O	C	C
2	O	C	O
3*	C	C	O
4	C	O	O

*SYNCHRONIZED CLOSING OF THE RUNNING CONTACTOR, R, IS REQUIRED.

Fig. 12 - Single-phase diagram illustrating capacitor-type starter and variables used in analysis.

(b) Dynamic Analysis

The dynamic analysis is performed with d- and q-axis orthogonal coordinates, the variables being obtained by transformation from instantaneous phase voltages and phase currents. The stationary reference frame is used in the interest of having the most sparsely filled matrix of operational impedances. The q-axis is fixed to the stator phase a axis and the q-axis is in quadrature leading the d-axis. The phase sequence is a-b-c. On these bases the matrix voltage equation of the induction machine is

$$\begin{bmatrix} v_{qs} \\ v_{ds} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + L_{ss}P & 0 & Mp & 0 \\ 0 & r_s+L_{ss}P & 0 & Mp \\ Mp & -Mp\theta_r & r'_r+L_{rr}p & -L_{rr}p\theta_r \\ Mp\theta_r & Mp & L'_{rr}p\theta_r & r'_r+L_{rr}p \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i'_{dr} \end{bmatrix} \qquad \ldots(11)$$

Equation (11) is obtainable directly from the work of Krause and Thomas, who have developed generalized equations, in d-q variables referred to an arbitrarily rotating reference frame, for the induction machine. The referenced work applies to a symmetrical three-phase machine having no saturation but otherwise conforming to the assumptions and idealizations used here. The introduction of saturation is postponed until the computer simulation used in verifying the criterion is considered. In obtaining (11) from the referenced generalized equations, the angular velocity of the arbitrarily rotating d-q axes is taken to be zero.

The shunt capacitor bank has the same terminal voltage as the motor stator. In the stationary reference frame of the d-q substitute variables, the capacitor voltages are

$$V_{qs} = \frac{W_e X_c i_{qc}}{p} = \frac{-W_e X_c i_{qs}}{p} \qquad \ldots\ldots (12)$$

and

$$V_{ds} = \frac{W_e X_c i_{dc}}{p} = \frac{-W_e X_c i_{ds}}{p} \qquad \ldots\ldots (13)$$

The second forms of (12) and (13) are valid when the motor is coasting and $i_{qL} = i_{dL} = 0$.

The dynamic matrix equation of the coasting system is obtained by eliminating Vqs and v_{ds} among (11), (12), and (13).

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (r_s + \frac{X_{ss}p}{W_e} + \frac{W_e X_c}{p}) & 0 & \frac{X_m}{W_e}p & 0 \\ 0 & (r_s + \frac{X_{ss}p}{W_e} + \frac{W_e X_c}{p}) & 0 & \frac{X_m}{W_e}p \\ \frac{X_m}{W_e}p & -X_m(\frac{W_r}{W_e}) & r'_r + \frac{X'_{rr}p}{W_e} & -X'_{rr}(\frac{W_r}{W_e}) \\ X_m(\frac{W_r}{W_e}) & \frac{X_m p}{W_e} & X'_{rr}(\frac{W_r}{W_e}) & r'_r + \frac{X'_{rr}p}{W_e} \end{bmatrix} \times \begin{bmatrix} i_{qs} \\ i_{ds} \\ i'_{dr} \\ i'_{dr} \end{bmatrix} \quad \text{.........(14)}$$

The characteristic equation of the system, found by requiring the determinant of the operational impedance matrix in (14) to vanish, is

$$a_6p^6 + a_5p^5 + a_4p^4 + a_3p^3 + a_2p^2 + a_1p + a_o = 0 \qquad \ldots(15)$$

where

$$a_6 = (X_m^2 - X_{ss}\,X'_{rr})^2 \qquad \ldots(16)$$

$$a_5 = 2w_e(X_{ss}\,X'_{rr} - X_m^2)\;(X_{ss}r'_r + X'_{rr}r_s) \qquad \ldots(17)$$

$$a_4 = w_r^2(X_m^2 - X_{ss}X'_{rr})^2 + 2w_e^2X'_{rr}X_c\;(X_{ss}X'_{rr} - X_m^2) \qquad \ldots(18)$$

$$a_3 = 2w_e\,w_r^2X'_{rr}r_s\;(X_{ss}X_{rr} - X_m^2)$$

$$+2w_e^3X_cr'_r\;(X_{ss}X'_{rr} - X_m^2)$$

$$+2w_e^3X_cX'_{rr}\;(X_{ss}r'_r + X'_{rr}r_s) \qquad \ldots\ldots\ldots(19)$$

$$a_2 = w_e^4 (X'_{rr})^2 X_c^2 + 2w_e^2 w_r^2 X'_{rr} X_c (X_{ss} X'_{rr} - X_m^2) \qquad \ldots (20)$$

$$a_1 = 2w_e^5 X_c^2 X'_{rr} r'_r + 2w_e^3 w_r^2 (X_{rr})^2 X_c r_s \qquad \ldots (21)$$

$$a_0 = w_e^4 w_r^2 (X'_{rr})^2 X_c^2 . \qquad \ldots (22)$$

Slight simplifications have been made in the foregoing coefficients. Twelve terms containing the factors r_s^2, $(r_r)^2$, $r_s r_r'$ and all higher degree resistance factors have been discarded.

The voltage stability can be determined from the characteristic equation. Instability corresponds to self-excitation. Routh's criterion has been applied to find the borderline of stability. The Routh array follows:

p^6	a_6	a_4	a_2	a_0
p^5	a_5	a_3	a_1	
p^4	$\frac{a_4a_5 - a_3a_6}{a_5} = b_1$	$\frac{a_2a_5 - a_1a_6}{a_5} = b_3$	a_0	
p^3	$\frac{a_3b_1 - a_5b_3}{b_1} = c_1$	$\frac{a_1b_1 - a_0a_5}{b_1} = c_3$		
p^2	$\frac{b_3c_a - b_1c_3}{c_1} = d_1$	a_0		
p^1	$\frac{c_3d_1 - a_0c_1}{d_1} = c_1$			
p^0	a_0			

In the left hand column of calculated terms, only the term c_1 has the possibility of changing sign. The borderline of stability has been found by equating c1 to zero. The borderline of stability which corresponds to a positive, real rotor speed is

$$X_c = X_{ss} \left(\frac{w_r}{w_e}\right)^2 \qquad \ldots.(23)$$

and self-excitation can be expected to occur when

$$X_c < X_{ss} \cdot \left(\frac{w_r}{w_e}\right)^2 \qquad \ldots\ldots (24)$$

Equation (23) defines the critical value of X_c and (24) is the criterion sought for self-excitation.

3.9 BRAKING UTILIZING SECONDARY CAPACITANCE EXCITATION (2)

With secondary capacitance excitation the system is shown in Fig. 13. in the system equal capacitors were connected via slip rings to the rotor of the induction motor. The capacitors perform a double role since they operate as power factor correction devices in the normal motoring mode of the SER system, Fig.13. The primary, stator windings of the machine were connected to a standard circuit consisting of a full-wave diode bridge, a smoothing choke and a full-wave controlled thyristor bridge. Current from the primary windings of the motor is rectified by the diode bridge and supplied to the thyristor bridge which operating in its invertor mode, transfers energy to the ac supply at a rate fixed by the thyristor firing-angle, the direct voltage at the invertor terminals and the supply voltage.

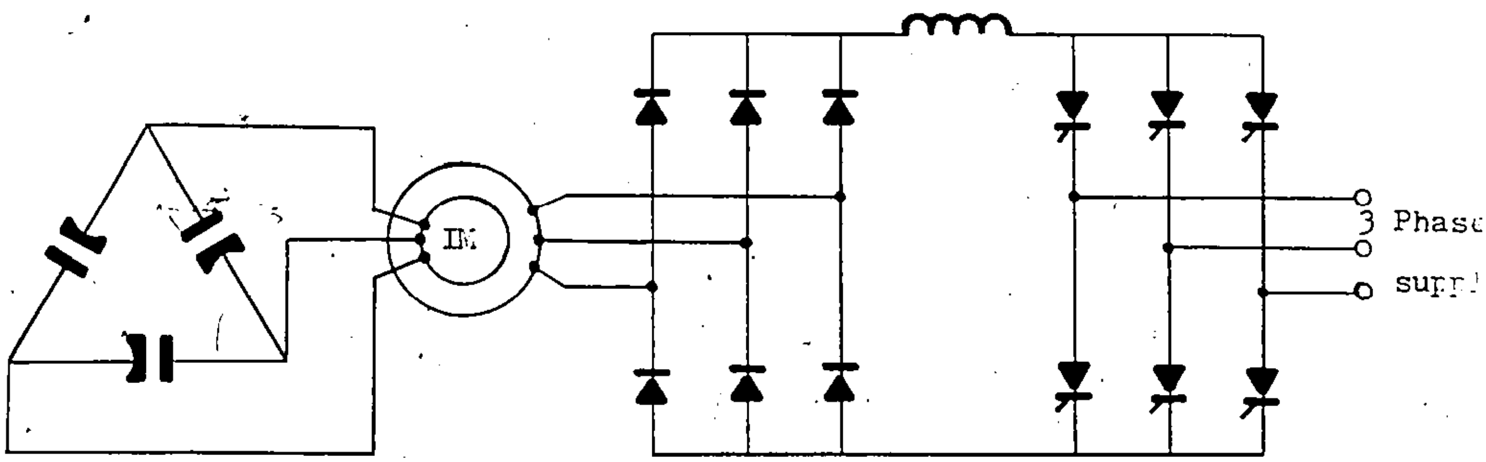


Fig.13 - Secondary balanced capacitor-excited induction motor with SER system in primary circuit.

3.9.1 Primary Windings of Induction Motor Short-Circuited (2)

(a) Equal Capacitors: Consider now the standard SER drive with switches, S, open, (Fig.7) the primary motor windings short-circuited and equal capacitors, connected across the secondary windings. Measured values of braking torque are shown in Fig.14, and are found to demonstrate symmetrical excitation of the motor.

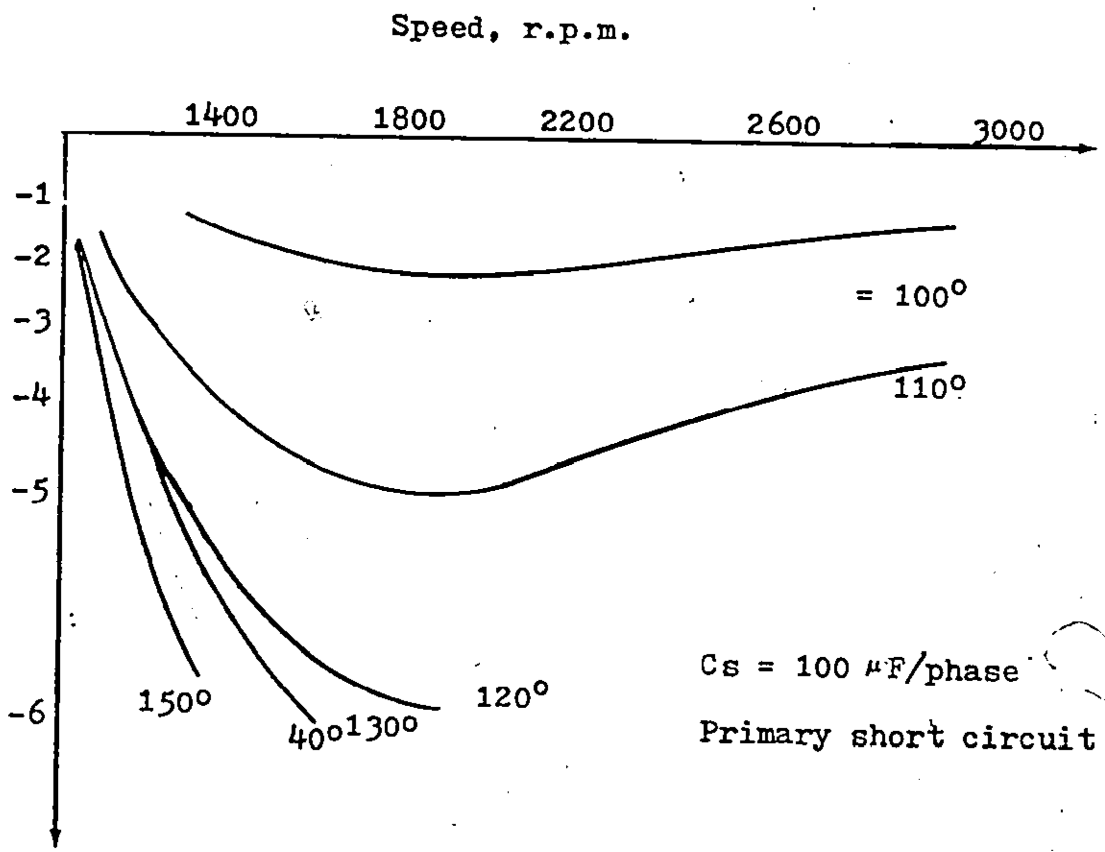


Fig.14 braking torque/sped performance with balanced secondary excitation.

The voltage appearing across the capacitors is limited by the voltage in the dc link. The invertor "internal voltage," in turn, depends on the supply voltage and the invertor firing angle. The degree of saturation of the machine depends mainly on the secondary emf and the speed. This suggests that the machine was operating unsaturated for low values of firing angle at most speeds and also for large firing angles at higher speeds.

For this mode of operation capacitance excitation occurs initially at a speed determined by the magnetization characteristic of the machine and the value of the capacitance.

Variation of invertor power with speed, for the same constant values of invertor firing angle, results in curves similar to those of Fig.14.[2] Much of the braking energy is extracted from the drive by the invertor and returned to the three phase supply. The braking torque, at a given speed, is directly controlled by the firing angle of the invertor which leads to the possibility of closed-loop control. The power factor of the system is that of the invertor which is a function of firing angle.

The displacement of the braking torque/speed characteristics (Fig.14) can be changed considerably by utilizing different values of balanced secondary capacitance. For lower values of capacitance the initial excitation occurs at a relatively higher speed, the general shape of the characteristics being unchanged.

(b) Unequal Capacitors:

With unequal secondary capacitors the braking torque/speed characteristics were found to be of the same form as those obtained with equal secondary capacitors as shown in Fig.15. With two capacitors, machine excitation builds up in a balanced manner in that all secondary voltages are equal in magnitude. Braking torque is produced initially at a higher speed than that obtained with equal capacitors of the same value. (2)

With only one secondary capacitor in the circuit, the machine exhibited a symmetrical buildup of excitation giving rise to braking torque characteristics similar in form to those of Fig.15. The speed at which excitation initially occurred was further increased, and the braking torque produced was in general decreased but remained a function of the invertor firing angle for a given machine speed.

For operation with short-circuited primary windings, therefore, a reduction in the number of capacitors or a

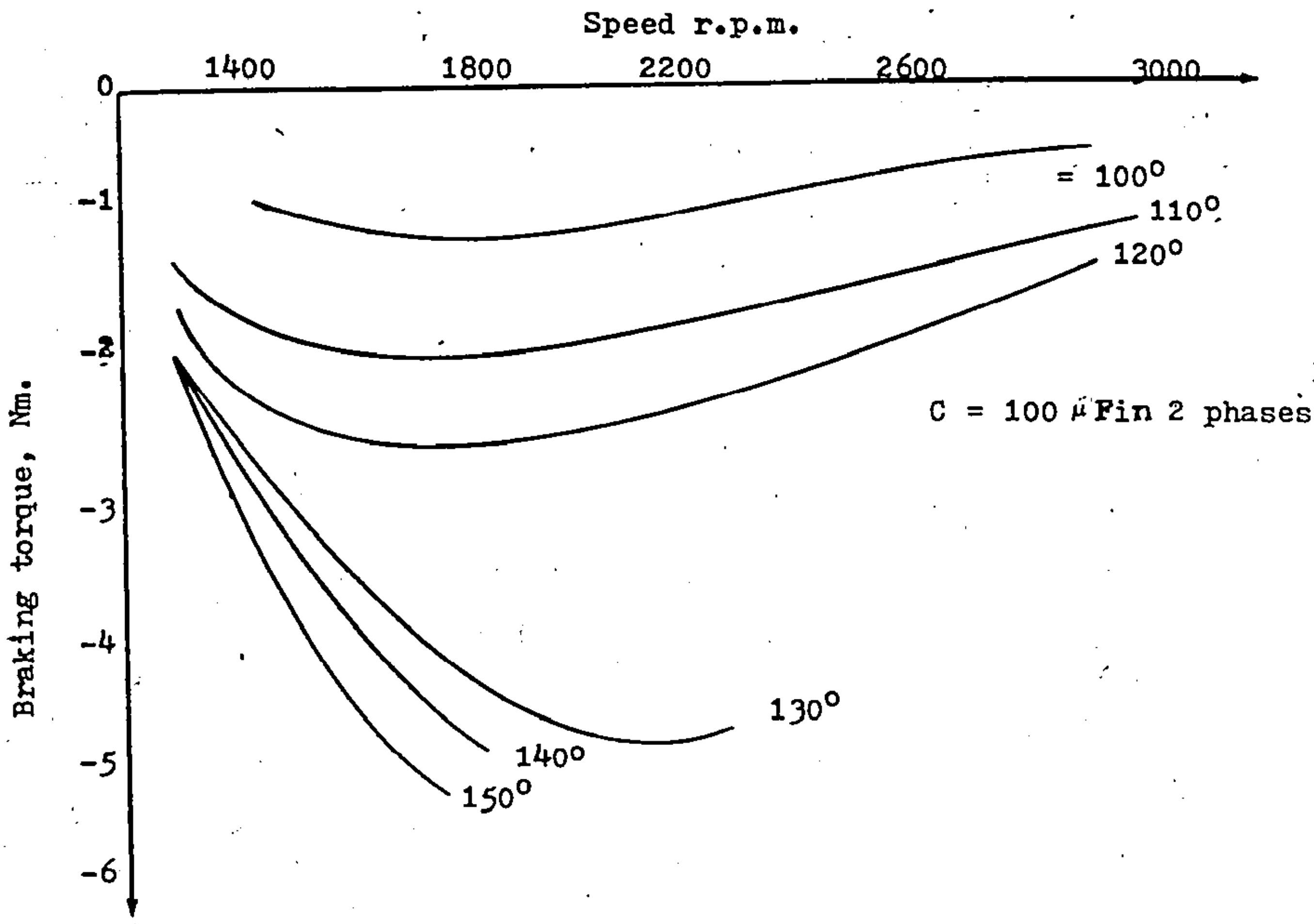


Fig. 15 - Braking torque/speed performance with un-balanced Secondary excitation.

reduction in the capacitance value resulted in a higher speed at which the initial excitation occurred but motor excitation was always symmetrical. (2)

3.9.2 Mains Supply to Primary of Machine (2)

(a) Equal Secondary Capacitance:

Consider operation of the standard SER drive (Fig.7) with equal capacitances connected across the secondary windings at the diode terminals. This secondary compensated SER circuit gives some measure of power factor correction . . . Also the addition of secondary capacitance results in greater torque, better speed regulation and displaces the torque curves towards the high speed region compared with uncompensated operation.

Steady-state braking is possible with this circuit with secondary compensation provided that the speed, thyristor firing angle and capacitor are of suitable values. The rectifier-invertor combination acts as a variable resistance in parallel with the secondary capacitance. Capacitance excitation of the machine gives rise to the braking torques produced.

(b) Unequal Capacitors:

Braking characteristics can be obtained with capacitors of unequal value in the secondary circuits. An investigation was made (2) with one, and two, of the capacitors removed from the normal delta connection. Using two capacitors only the variation of braking torque with speed, for constant invertor firing angles, is shown in Fig. 16 Symmetrical and unsymmetrical modes of operation are both in evidence for invertor firing angles of 150, 140 and 130 degrees, the transition occurring with change of speed. No braking torque was obtained below one-half synchronous speed. A reduction in capacitor value resulted in a shift of the curves to higher speeds.

The unsymmetrical excitation occur near to one-half speed and are of greater gradient than the symmetrical excitation characteristics which are dispersed over the higher speed range.

With a single capacitor in the secondary circuit the torque/speed curves were similar to those of Fig.16. The unsymmetrical mode of operation was much less in evidence then and the the curves for symmetrical excitation, in general, were displaced to a higher speed compared with corresponding results obtained with two or three capacitor banks.

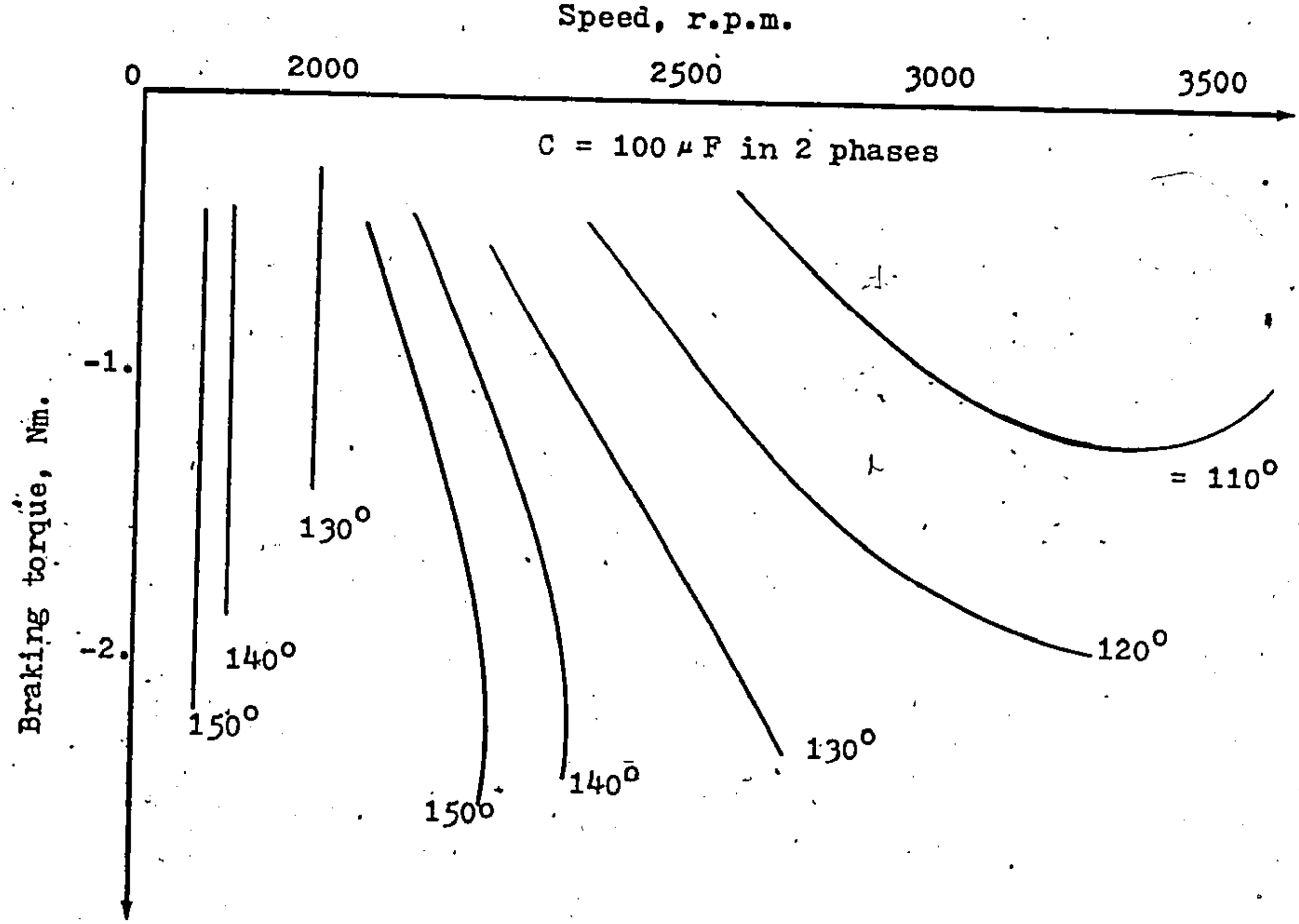


Fig. 16. Braking torque/speed performance with mains primary excitation and unbalanced secondary capacitor excitation.

3.10 Speed-Torque characteristics calculations: (2)

A per-phase equivalent circuit for an induction motor with primary capacitor excitation is given in Fig.17.

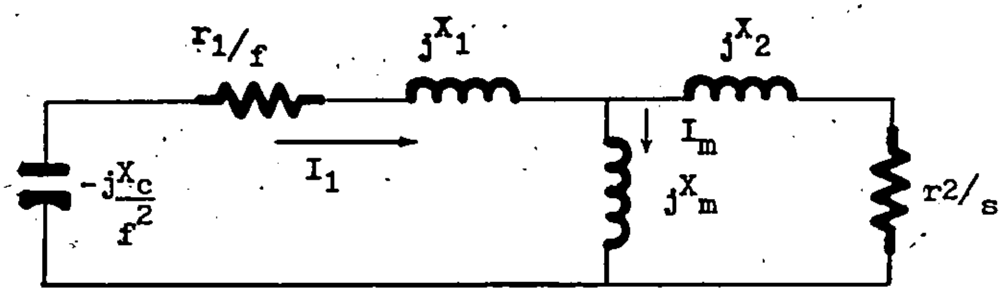


Fig. 17. A per phase equivalent circuit for an induction motor with primary capacitor excitation.

This is derived by standard methods and all component values are those measured by established tests at a frequency of 50 Hz and referred to to the primary side.

From a consideration of the equivalent circuit and the defined current flow:

$$I_1\ (r_1/f + jx_1 - j\ (X_c/f^2)) + I_m j x_m = 0 \qquad (25)$$

$$I_2\ (r_2/s + js_2) - I_m j x_m = 0 \qquad (26)$$

$$I_1 - I_2 = I_m \qquad (27)$$

By combining the above three equations to eliminate I_m and I_2:

$$r_1 r_2/fs - (x_1 + x_m - x_c/f^2)\ (x_2 + x_m) + j(r_2/s)$$
$$\cdot (x_1 + x_m - x_c/f^2) + j(r_1/f)\ (x_2 + x_m) + x_m^2 = 0 \quad (4)$$

By equating real parts of the above equation:

$$r_1r_2/fs - (x_1 + x_m - x_c/f^2)\ (x_2 + x_m) + x_m{}^2 = 0 \qquad (28)$$

By equating imaginary parts of equation (A.4):

$$(r2/s)\ (x_1 + x_m - x_c/f^2) + (r_1/f)\ (x_2 + x_m) = 0 \qquad (29)$$

By combination of equation (28) with (29), x_m can be eliminated:

$$s^2(r_1s_2{}^2f^3) + s(-2_{r2r1f}{}^2x_c + r_2x_1{}^2f^4 + r_2x_c{}^2 + r_1{}^2r_2f^2) + r_2{}^2{}_{r_1}f^3 = 0. \qquad (30)$$

Equations (25) and (26) are in terms of the vector quantities of the currents and can be converted to quantities involving only the magnitudes of the current as follows:

$$I_1 = \frac{I_mX_m}{((r_1/f^2) + (x_1 - x_c/f^2)^2)^{1/2}} \qquad (31)$$

$$I_2 = \frac{ImXm}{((r_2/s)^2 + x_2{}^2)^{1/2}} \qquad (32)$$

Also it is obvious that:

$$T = I_2{}^2 r_2/Nos \qquad (33)$$

$$N = (f - s)\ No \qquad (34)$$

$$V_c = I_1(x_c/f) \qquad (35)$$

Rewriting equation (29) gives

$$f^2 r_2(x_1 + x_m) + fsr_1(r_2 + x_m) - r_2 x_c = 0$$

Let

$$A = (x_1 + x_m) r_2$$

$$B = r_1(x_2 + x_m)$$

$$C = -r_2 x_c$$

then

$$f^2 A + fBs + C = 0 \qquad (36)$$

Rewriting equation (28)

$$f^2 s((- x_1 - x_m)(x_m + x_2) + x_m^2)$$

$$+ f r_1 r_2 + s x_c (x2 + x_m) = 0$$

Let

$$D = (-x1 - x_m)(xm + x_2) + xm^2$$

$$E = r_1 r_2$$

$$F = x_c (x_2 + x_m)$$

then

$$f^2 sD + fE + sF = 0 \qquad (37)$$

Combination of equations (36) and (37) gives

$$f = - \frac{(FA - CD)s}{(EA - s^2 BD)} \qquad (38)$$

Substitute back in equation (36) to eliminate f

$$s^4 (B^2DF) + s^2(AF - CD)2$$
$$- BE(AF + CD) + ACE^2 = 0 \qquad (39)$$

It is very convenient that the above equation contains no quantities of order s^3 and hence will make solution of the equation easy. It will also be noted that if all the circuit values are defined then the value of slip s can be obtained, and the frequency of excitation f has been eliminated.

It has been stated that the total secondary resistance can be calculated from the initially defined conditions. The only variable components involved in equation (39) are the slip s and xm the magnetizing reactance. Because the machine saturates, the value of xm is variable; hence equation (39) could be restated as

$$s = f_1(x_m). \qquad (40)$$

From the initial conditions it is possible to calculate the value of Vm from equation (32) if the slip is known. A polynomial function was used in which Im is given for values of Vm , and hence the value of magnetizing reactance xm can be calculated.

Thus equation (32) allows one to calculate the value of Xm if the slip s is known and hence can be restated as

$$X_m = f_2(s). \qquad (41)$$

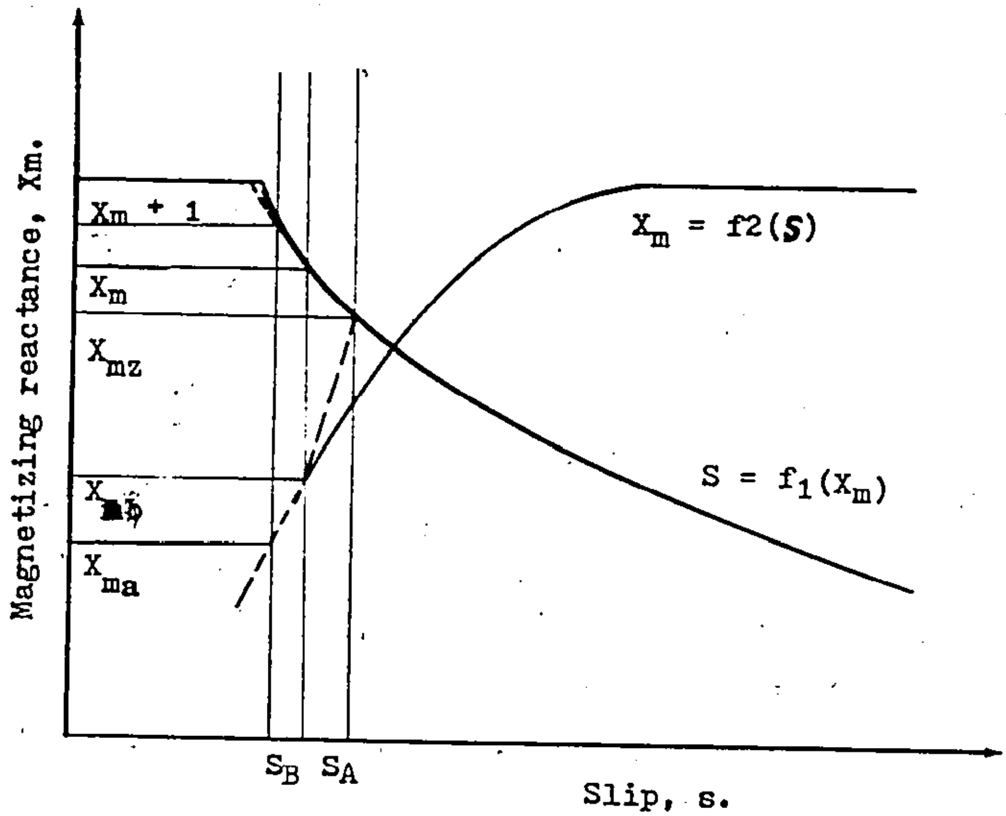


Fig.18 Approximate form of functions f_1 and f_2.

The form of the two equations (40) and (41) is shown in Fig.18 for defined values of invertor firing angle and dc. link current. The first method of calculation tried, namely taking any value of Xm, substituting in equation (39) to obtain a value for s, and using this latter value in equation (32) to obtain a new value of Xm, proved to be unusable as the method does not converge.

The successful method involved using equation (39) to calculate two values of slip s_A and s_B for two close values of magnetizing reactance Xm and X_m+1. The two values of slip s_A and s_B were substituted in equation (39) to give two values of magnetizing reactance Xm and Xmb. From these sets of values, tangents to the two curves are known and the Xm, value at their intersection can be calculated. This is shown in Fig.18 and the new value Xm, was used in the next calculation. It was found that the required solution was found in less than 6 iterations.

Once the value of slip and magnetizing reactance are known the frequency of excitation, primary current, torque, speed and capacitor voltage can be calculated from equations (38), (33), (34) and (35), respectively.

3.11 Effect of motor magnetic saturation on the braking operation: (3)

The equivalent circuit of an induction motor under excitation conditions differs from the one under normal operation conditions in that there is no applied supply voltage. Instead a capacitor is connected across the terminals where the voltage is normally applied.

In this circuit, the varying frequency parameters are referred to the constant frequency of the supply for which the motor is designed. For the equivilant circuit shown, both voltage and impedances values must be multiplied by the excitation frequency. Currents, however are correct values at excitation frequency as they are equal to: Voltage/impedance and therefore the multiplication factor disappears.

In the equivalent circuit fig.19 consider the two loops, one the stator circuit through the magnetising reactance and the other, the rotor circuit through the same magnetising reactance. As the voltages in these loops are zero, they give the following equations.

$$I_1\left\{\frac{r1}{f} + j\left(x1 - \frac{Xc}{f^2}\right)\right\} + (I_1 + I2)\, jxm = 0 \qquad (42)$$

$$I_2\left(\frac{r2}{f-n} + jx2\right) + (I_1 + I_2)\, jxm = 0 \qquad (43)$$

where $n = \dfrac{\text{Speed}}{\text{Synchronous speed}}$

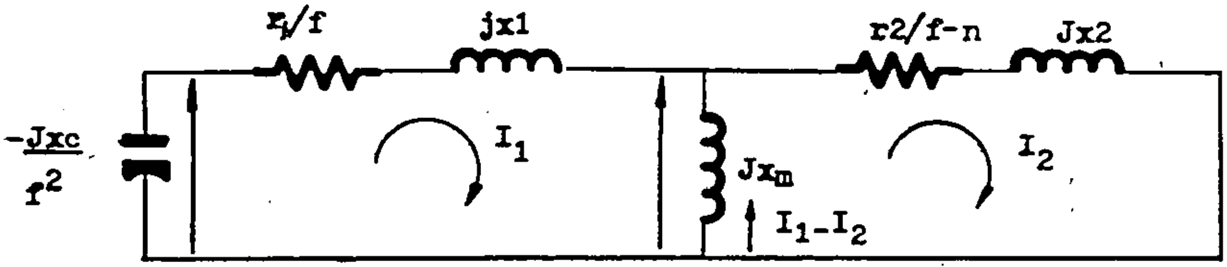


Fig.19 Equivalent circuit for an induction motor during capacitor braking (3)

Elimination of I_1 and I_2 from (41) and (42) above gives

$$\left\{\frac{r1}{f} + j\left(x1 + xm - \frac{Xc}{f^2}\right)\right\}\left\{\frac{r2}{f-n} + j\,(x2 + Xm\right\} + X_m^2 = 0 \quad (43)$$

Real and imaginary parts of (43) must be individually zero to satisfy the equation. Therefore

$$\frac{r1}{f} \times \frac{r2}{f-n} - \left(x1 + xm - \frac{xc}{f^2}\right)\ (x_2 + xm) + x2m = 0 \qquad (44)$$

and

$$\frac{r1}{f}(x2 + xm) + \frac{r2}{f-n}\left(x1 + Xm - \frac{Xc}{f^2}\right) = 0 \qquad (45)$$

The value of magnetising reactance Xm, being dependent on the saturation of the magnetic circuit, is variable over a wide range during self excitation. The value may vary from normal to one tenth of normal. Therefore Xm can be eliminated from equations (44) and (45) to obtain a relation between xc and the other machine constants for chosen values of 'f' and 'n'.

This can be written in the form

$$xc = f^2 x1\left[1 + \sqrt{\left(\frac{r1}{r2} x \frac{n-f}{f}\right)\left\{\left(\frac{\frac{r2}{x1}}{n-f}\right)^2 x \left(1 - \frac{r1}{r2} x \frac{n-f}{f}\right) - \left(\frac{X2}{X1}\right)^2\right\}}\right] \quad ...(46)$$

The value of xm can be found from equation (45)

$$\frac{xm}{x1} = \frac{\left(\frac{r1}{r2} x \frac{n-f}{f}\right) \frac{x2}{x1} - \left(1 - \frac{xc}{X_1} x \frac{1}{f2}\right)}{1 - \frac{r1}{r2} x \frac{n-f}{f}} \quad ...(47)$$

From equation(46). $1-\frac{r1}{r2} x \frac{n-f}{f}$ must be positive

to make the quantities under the square root positive so that a real value of xc is obtained. This condition is also indicated in equation (7). From this the limiting value of f is obtained by the relation

$$f > \frac{n}{1 + \frac{r2}{r1}}$$

Knowing the value of xm/X1 for any chosen values of n and f, and therefore xm, the value of V_g can be obtained from the magnetising curve of the motor and hence the currents and torque can be found.

T = torque is synchronous watts = $3 x \frac{r2}{n-f} x I_R^2$

3.12 LOSSES DURING BRAKING

Since braking depends on the expenditure of energy, it is useful to consider first the motor losses for changes of speed, during acceleration or deceleration.

Using the new concept 'power loss per unit of torque, W/T, losses under different conditions can be quickly compared. The normal equivalent circuit of an induction motor is shown in fig.20(a) but as the magnetising current during braking forms only an insignificant part of the total loss it can be disregarded. The circuit thus becomes as in fig.20 (b), and W, which equals the copper loss per phase, is given by

$$W = I^2_R (r1 + r2) \text{ watts}$$

where I_R is the rotor current and r1 and r2 are the primary and secondary resistances, all referred to the primary. T, the torque per phase, is given by

$$T = \frac{1}{ws} x\ I_R^2 \ x \ \frac{r_2}{S} \qquad \text{N.m}$$

where S is the slip, Therefore

$$\frac{W}{T} = S \ x \ \left(1 + \frac{r1}{r2}\right) X \ W_s$$

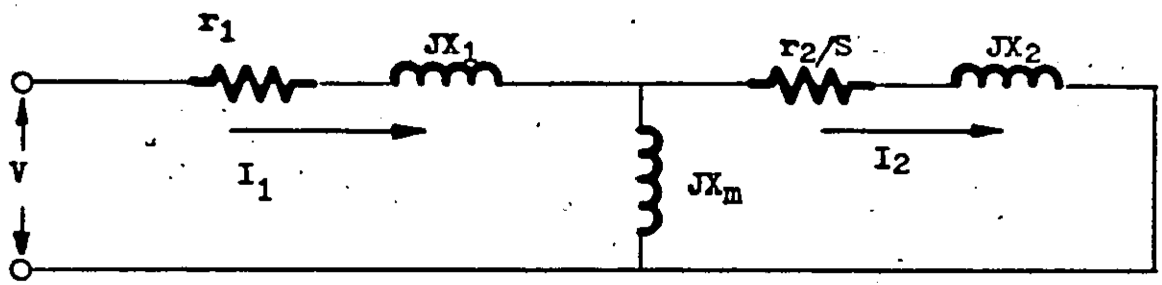


Fig 20 (a) Equivalent circuit of an induction motor (Normal circuit)

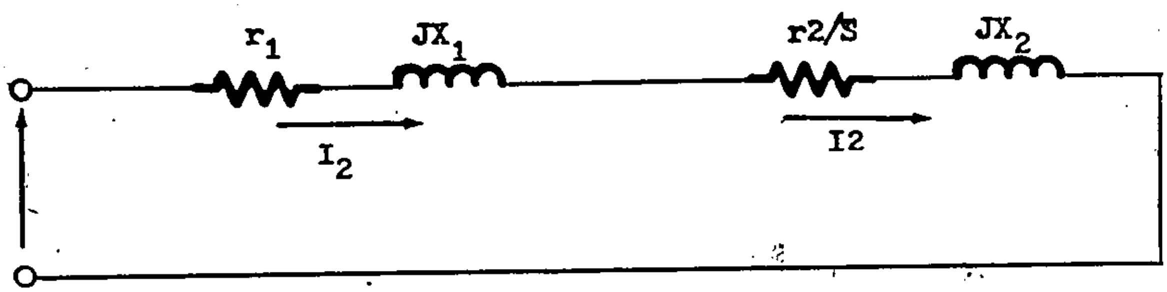


Fig. 20 (b) The equivalent circuit during acceleration

The stator loss can be obtained by multiplying the rotor loss by r1/r2 and as the latter only need be considered, W/T = S. Since this is linear, the average loss per unit torque between any two speeds with slips of S1 and S2 is given by

$$\frac{W}{T} = \frac{S1 + S2}{2}$$

But

$$T = \frac{M \times Ns}{7.04}$$

where M is the torque in lb ft, and Ns is the synchronous speed. Therefore the average rotor loss W is given by

$$\frac{S1 + S2}{2} \times \frac{M \times Ns}{7.04} \qquad (48)$$

In equation (48), the torque is calculated at the two values S1 & S2.

It is known that the time t in seconds to produce a change of speed of Nrev/min for an inertia mR2 in lb ft^2 is given by

$$t = \frac{mR^2 \times N}{307.5 \times M} \qquad (49)$$

Multiplying (48) by (49), the rotor loss, Wt, is obtained as follows

$$Wt = (S1 + S2) \times mR^2 \times N \times \frac{Ns}{4330} \text{ watt seconds} \qquad (50)$$

Equation (50) shows that for a required speed change, N, the rotor loss is directly proportional to the sum of the slips. Thus during acceleration S_1 at standstill = 1, S_2 at synchronous speed = 0 and therefore S1 + S2 = 1 and the loss is proportional to 1. During deceleration by 'plug reversing' S1 at synchronous speed = 2, S_2 at standstill = 1 and S1 + S2 = 3; the loss, being proportional to 3, is therefore three times the loss during acceleration.

If retardation were carried out by changing the frequency of the supply current, the losses would be much lower, as may be illustrated by taking the case of a 6-pole, 50c/s motor. Suppose that a speed reduction to 200rev/min is required (i.e. N = 800). By plug reversing S1 = 2 and S2 = 1.2 and the rotor loss is given by

$$\text{rotor loss} = mR^2 \times 3.2 \times 800 \times \frac{1000}{4330} \text{ watt seconds}$$

By frequency changing, the values are as shown in table 2, so that the rotor loss is given by

$$\text{rotor loss} = mR^2 \times 0.2 \times 800 \times \frac{1000}{4330} \text{ watt seconds}$$

Thus the loss by plug reversing is 3.2/0.2 or sixteen times that by frequency changing.

To achieve braking by the use of a frequency changer of the usual type would be very expensive, but it has been found that the introduction of capacitance into the circuit during the braking period gives rise to a similar effect, for the frequency of the self-excitation current is always slightly lower than the value corresponding to that of the motor speed, and continuously decreases as the speed falls. At very low speeds, when the self-excitation frequency becomes the same as the speed value, the braking effect disappears but, by the choise of suitable capacitor value, this may be as low as 10% of full speed.

Equation (50) for rotor loss indicates that the loss during acceleration and retardation is entirely independent of the machine design, and hence it cannot be altered by change of applied voltage. In a slipring mötor the main part of these losses can be transferred to external rotor circuit resistances. Stator loss is controlled by change of rotor resistance, since it depends on the ratio r_1/r_2.

3.13 BRAKING TIME AND TEMPERATURE RISE (6)

Knowing the load torque and the motor torque during the braking period, the resultant braking-torque curve can be drawn, over a convenient number of speed ranges. This curve may be assumed to be linear, and by calculating the average braking torque over a particular speed range, the time required for the reduction of speed can be calculated. The sum of all these time intervals gives the total braking time. Adopting this procedure, the braking times for the four methods under consideration have been calculated and given in table 3. By plotting a relation between the calculated power input to the machine and losses for the following methods of braking: a) Plugging, b) Capacitor braking c) Capacitor reactor braking, the difference between the ordinates of the two carves gives the power absorbed from the rotating masses. Table 2, shows the total energy loss, the average energy loss and the braking time, gives a comparative idea of the temperature rises which may be expected with the given three methods of braking with a constant load torque of 1000 synchronous watts.

TABLE 2. Calculated Energy Losses (6)

Method of braking	Total Energy Loss	Average Energy Loss	Braking Time
1) Plugging	KW. sec.	KW. Sec.	Sec.
2) Capacitor braking (-J80 Ohm)	6.56 11.996	4.63 1.49	1.42 8.05
3) Capacitor Reactor Braking (-J80 Ohm & + J40 Ohm)	2.108	0.668	3.16

TABLE 3

BRAKING TIMES

Load torque	Method of braking	Braking time	
		Calculated	Test
		sec	sec
Friction and windage losses only	Main switch open	-	39
	Plugging	2.04	-
	Capacitor braking (-j80 ohms)	∞	∞
	Mains cut off during the driving torque range, then capacitor braking (-j80 ohms)	19.87	17.8
	Induction generator during the driving torque range, then capacitor braking (-j80 ohms)	10.27	10
	Capacitor-reactor braking (-j80 ohms and j40 ohms)	11.3	10.5
1000 synchronous watts (constant)	Main switch open	-	19
	Plugging	1.42	-
	Capacitor braking	8.05	7.6
	Capacitor-reactor braking	3.16	3.0

The parameters of the induction motor are:

3h.p., 400 volt, 50c/s, 1440r.p.m., 3-phase delta-connected squirrel-cage motor, which was available, was chosen. Its measured constants were $Zm = 35 + j418$, $Z_1 = 8.28 + j20$, $Z_2 = 10.1 + j20$, where Zm is the magnetizing impedance, and Z_1 and Z_2 are the leakage impedance of the stator and rotor windings.

3.14 <u>Speed-Time Relation</u>

The net friction or overhauling torque of the load as a function of speed and the total WR^2 of the rotating system must be known. Then with the braking torque obtained the net retarding torque T_r will be

$$T_r = T = T_L$$

where T_L is the torque produced at the shaft of the motor by the driven machine. Tr will be represented by a curve and in general will be a function of speed alone.

The usual equation relating T_r and time is

$$(\text{Average})\ Tr = \frac{WR^2N}{307.8(t)}$$

where N is the change in speed occurring in t seconds.

In better form

$$T_r = \frac{WR^2}{307.8} \cdot \frac{dN}{dt}$$

expresses the torque as a function of the rate of change of speed.

Solving the foregoing equation for time t,

$$t = \frac{WR^2}{307.8}\int_{N_1}^{N} \frac{1}{T_r}\, dN$$

the integration can be made by a graphic tabular process to obtain the time t for speed to change from N_1 to any other value N.

4.0 PRACTICAL EXAMPLE:

This section deals with the practical problem of designing a capacitor braking system for a typical 5 HP induction motor to bring the motor from 1700 rpm to 1200 rpm and developing a maximum braking torque without exceeding 150% of the rated current, with the parameters given as:

Primary Leakage Inductance	$L_s = 2.22$	mhenries
Secondary Leakage Inductance	$L_r = 2.20$	mhenries
Magnetizing inductance	$M = 66.84$	mhenries
Primary resistance	$R1 = 0.44$	ohms
Secondary resistance	$R2 = 0.708$	ohms
Number of pole-pairs	$n = 2$	
Rated frequency	$f = 60.0$	Hz
Rated RMS Voltage	$V_{ph} = 127.0$	volts
Rated output torque	$Te = 17.37$	Nm
Total Viscous friction	$F_T = 0.0014$	Nm sec/rad
Total (Motor+Load) Inertia	$J_T = 0.1$	Kg m^2
Rated speed	$N = 1700$	rpm.

4.1 TYPICAL SOLUTION

In this section all computational steps will be explained with the corresponding formulas in order to size the capacitors bank if the maximum allowable stator current is given. Taking into considerations all the variables involved, (no approximations or assumptions have been made) including the motor magnetic saturation. The equivilant circuit from which these formulas are obtained is shown in Fig. 17 .

Computational Steps:

(1) Assume the value of the excitation frequency (fex) in the range of 59 Hz to 40 Hz in steps each of 0.1 Hz.

(2) For each assumed value calculate the following parameters

$X_1 = 2\pi \, fex \, L_1$ $(L_1 = 2.22 \text{ m H})$ $X_1 = 0.0139487 \, fex$

$X_2' = 2\pi \, fex \, L_2$ $(L_2 = 2.20 \text{ mH})$ $X_2 = 0.013823 \, fex$

(3) a. The values of $Xc = \dfrac{1}{2\pi \, fex \, C}$

$Xm = 2\pi \, fex \, M$

$f = fex/60$ Hz

$S =$ Slip at fex

Can be calculated accordingly by solving the four following equations simultaneously.

(i) $Xc = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ (51), Where: $a = \dfrac{R2}{f^2(SfR1 + f^2R_2)}$

$$b = \frac{1}{f^2(SfR_1 + f^2R_2)}\left[X_2(SFR_1 + f^2R_2) - X_1R_2f^2 - R_2X_2f^2 - SfR_1X_2 - X_1R_2f^2\right]$$

$$c = \frac{f_5}{SfR_1 + f^2R_2}(SR_1X_1X_2 + R_2X_1^2f + SR_1X_2^2 + X_1X_2R_2f) + \frac{R_1R_2 - X_1X_2}{f_s}$$

(ii) $$Xm = \frac{R2Xc - Sf(R_1X_2) - f^2(R_2X_1)}{Sf(R_1) + f^2(R2)} \quad (52)$$

(iii) $$f = \frac{S\{[Xc(X2+Xm)][R2(X_1+Xm)] - [(-R2Xc)(-X1Xm - X1X2 - XmX2)]}{R_1R_2^2(X1+Xm) - S^2\{R_1(X2+Xm)[-X_1Xm - X_1X_2 - Xm X_2]} \quad (53)$$

(iv) $$\begin{aligned} &S^4\left\{[R_1^2(X_2^2 + Xm^2 + 2X_1X_m][Xc(X_2+Xm)][-X_1Xm - X_1X_2 - X_mX_2]\right\} \quad (54)\\ &+ S^2\left\{[R_2(X_1+X_m)][Xc(X2+Xm)] + R_2Xc(-X_1Xm - X_1X_2 - X_mX_2)\right\}\\ &- [R_1^2R_2(X2+Xm)][XcR2(X1+Xm)(X2+Xm) + R2Xc(X_1Xm + X_1X_2 + XmX2)]\\ &- R_1X_cR_2^4(X_1 + Xm) = 0 \end{aligned}$$

(b) With the value of f $fex = f \times 60$

The value of fex checked against the assumed value, if it does not correspond, reasume a value of ($fex + \Delta fex$) and the previous step has to be repeated until the exact fex is obtained.

(4) Speed N is calculated at each value of fex using the formula: $N = \frac{60\ fex}{P}$ where $P = 2$

(5) Calculate Im

$$IM = \sqrt{\frac{Te\ W}{3R_2M^2}\left[\left(\frac{R2}{W}\right)^2 + (L2)^2\right]} \quad (55)$$

where T = 17.37 N.m

(6) Calculate I_1

$$I_1 = \frac{I_m\ X_m}{\sqrt{\left(\frac{R1}{f}\right)^2 + \left(X_1 - \frac{Xc}{f2}\right)^2}} \quad (56)$$

(7) Calculate I_2

$$I_2 = \frac{Im\ Xm}{\sqrt{\left(\frac{R2}{S}\right)^2 + (X_2)^2}} \quad (57)$$

(8) Calculate the capacitor voltage Vc (58)

$Vc = I_1\ (Xc/f)$

(9) Calculate the braking torque Tnr

$$Tbr = \frac{{I_2}^2 R_2}{fex\ S} \quad (59)$$

Check; Maximum Tbr at $I_1 \leq 16.088$ Amp (150%IN)

where IN = 10.725

Caloulated from the equivilant circuit of the induction motor without capacitor considered.

If the value of $I_1 > 16.088$ Amp. The value of capacitance C has to be decreased and calculations steps have to repeated from step No.1.

5.0 CONCLUSION

Capacitors having reactances comparable with the short circuit impedance of the induction motor were introduced into the motor circuit at the start of the braking period and the motor terminals were short-circuited at a suitable point during the fall of speed.

At this point maximum use could be made of the magnetic field which had been built up by self excitation through the capacitors, and a high braking torque over wide speed range could be obtained. The machine can be excited by placing capacitors in the primary or secondary windings of the motor.

With primary capacitance excitation, braking torque is controlled wither by using series or parallel resistors as a load or by using a rectifier-inverter combination which acts as a variable resistance. Braking torque is then a linear function of d.c. link current.

Braking torque is also produced by capacitance excitation due to secondary side capacitors. Equal capacitance per phase can result in balanced excitation and then high speed braking results. Also braking is possible with unequal secondary side capacitors. Also braking torque is produced with or without primary supply.

Again, variations of braking torque at high speeds is achieved by the use of resistance in series or parallel with the capacitors.

It will be clear from what has been said that the methods of braking described in this article represent a major advance in electrical technology. Since progress in one field is often reflected by advances in others, it is confidently expected that more and more sections of industry will come to benefit from the continued exploitation of this development.

6.0 REFERENCES

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6.1 Survey of the existing literatures:-

Brief description of the references given in section 6.0 will be given respectively.

(1) Introduction of the capacitor braking method have been explained together with two design methods, these are:

(a) Braking by the use of two capacitors connected in the three phase supply line.

(b) Braking by capacitor-rectifier-resistor combination.

(2) The paper describes the theory for capacitor excitation of an induction motor either by primary or secondary capacitance excitation. Performance characteristics calculation method have been outlined together with a speed torque characteristics for a 2HP induction motor.

(3) The theory of capacitance braking is described under the following headings:

(a) Losses during braking.

(b) Self excitation with capacitance with, or without controlling resistors.

(c) Magnetic braking using capacitor-resistor-rectifier circuit.

(d) Use of capacitor braking followed by the simultaneous use of magnetic and d.c. injection braking.

(4) D.C. dynamic braking of induction motor with secondary capacitors in series or parallel with resistors have been explained in this paper.

(5) The paper has introduced a detailed comparison of different braking methods. Analysis of the equivilant circuit during capacitor braking, losses and speed time relations have been explained as well.

(6) This paper applies the method of symmetrical components to the current voltage relations whilst a series capacitor is introduced in one of the supply lines of a three phase induction motor. The results have been given together with speed-torque characteristics.

(7) This paper describes the method of sizing the capacitance that can be connected in parallel with an induction motor and static load combination without self excitation.

(8) Transients in induction machines with terminal capacitors have been explained in detail. Analysis of the capacitor braking scheme, digital computer program and test results have been given.

(9) Electrical braking of a static slip-power recovery drive is the subject of this paper. There exists a similarity of the design criteria to the subject explained in paper no.2

(10) Application of self-excitation by primary capacitor to the induction motor for starting purposes have been explained in this paper.
Dynamic analysis using d & q axis to size the capacitors used together with test results is explained.